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DYNAMICAL AND ELECTRICAL PROBLEMS WHOSE SOLUTIONS SATISFY SCHRÖDINGER'S WAVE EQUATION.

BY

A. BRAMLEY, Ph.D.

Bartol Research Foundation.

BARTOL RESEARCH FOUNDATION
Communication No. 19.

IN THIS paper we shall attempt to prove a number of interesting relations which exist between the periodic solutions of Schrödinger's wave equation and the solutions of certain problems in mechanics, especially with regard to the Bohr-Sommerfield quantum mechanics, and also some potential problems connected with standing waves. For the purpose of brevity and clearness, these relationships will be expressed in the form of theorems.

THEOREM I.

Solutions of the equations of motion of an electric particle, written in the relativity form, in the case where the variables are separable, which satisfy the equation of continuity, also satisfy Schrödinger's wave equation.

There are one or more sets of differential equations which any distribution of electric charge in motion must satisfy, the first one which we shall consider is the equation of continuity which can be expressed in a rectangular coördinate system as follows:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0,$$

where ρ is the density of charge at any point and u, v and w the components of the velocity vector at the same point.

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In general, for any coördinate system whose linear element is

$$ds^2 = g_{\alpha\beta}\, dx^\alpha\, dx^\beta$$

where the repeated Greek indices are summed over the range from one to four and where the first three coördinates x^1, x^2 and x^3 represent the spatial coördinates and x^4 the time coördinate, the equation of continuity assumes the form

$$(Ga^\alpha)_\alpha = 0 \qquad (1.1)$$

where G is a scalar quantity usually taken as the charge density and a^i the contravariant components of the velocity vector in the x^i direction, and where the operation of covariant differentiation of the vector, whose contravariant components are R^1, R^2, R^3, and R^4 with respect to x^i, is given by the equation

$$(R^i)_j = \frac{\partial R^i}{\partial x^j} + \Gamma^i_{j\alpha} R^\alpha .$$

The quantities $\Gamma^i_{j\kappa}$, the three index symbols of Chistofell, are defined in terms of the g's by the following series of relations,

$$\Gamma_{\alpha\beta,\,i} = \frac{1}{2}\left(\frac{\partial g_{\alpha i}}{\partial x^\beta} + \frac{\partial g_{\beta i}}{\partial x^\alpha} - \frac{\partial g_{\alpha\beta}}{\partial x^i}\right)$$

$$g^{\alpha i} g_{\alpha j} = \delta^i_{\ j} \qquad \delta^i_{\ j} = \begin{matrix} 0 & i \neq j \\ 1 & i = j \end{matrix}$$

and

$$\Gamma^i_{j\kappa} = \Gamma_{j\kappa,\,\lambda}\, g^{\lambda i},$$

which completely determine the $\Gamma^i_{j\kappa}$'s in terms of the g_{ij}'s and their partial derivatives.

What the other set or sets of differential equations may be which govern the motion of a distribution of electric charge, we cannot very well at the present time say. Although such considerations have no definite bearing on the proof of the above-enunciated theorem, they are of importance in any physical interpretation of the results obtained. For the present purposes, however, we shall assume that the motion of the centre of inertia of the electrical system must be such as to satisfy the following set of equations:

$$(\mu Q^i)_\alpha\, Q^\alpha = \frac{2\pi i \mu}{h}\, G_{\alpha\beta}\, Q^\alpha\, Q^\beta\, Q^i \qquad (1.2)$$

$$i = 1 \cdots 4$$

$$Q^i = \frac{dq^i}{ds},$$

where $i = \sqrt{-1}$

h = Planck's constant

and μ = the mass of the system which we shall regard as constant during the type of motion we are considering.

This set of equations represents the motion of a mass particle in a Riemannian space with linear element

$$ds^2 = g_{\alpha\beta}\, dx^\alpha\, dx^\beta$$

under the influence of a retarding force whose components are proportional, respectively, to the velocity components in the same direction. They can also be regarded as the geodesics in a Weyl space with the above linear element but with the gauge factor λ chosen in such a manner that the vector ϕ_i is proportional to the momentum of the particle in the same direction

$$\phi_i = \frac{\partial \log \lambda}{\partial x^i}. \tag{1.3}$$

The solutions of equation 1.2 have been shown by the author to lead to solutions identical with those obtained by the quantum mechanics of Bohr and Sommerfield for the problem of the hydrogen atom provided the first integral of 1.2 is periodic when the variables are separable.

Restricting the consideration of equation 1.2 to the case where the variables are separable, *i.e.*,

$$Q^i = \frac{\partial \psi}{\partial x^i},$$

where $$\psi = \psi_1 (x^1, x^2, x^3) + \psi_4 (x^4),$$

$\psi_4 (x^4)$ being a function of x^4 only, the equations of motion take the form

$$\left(\mu \frac{\partial \psi}{\partial x^i}\right)_\alpha \frac{\partial \psi}{\partial x^\alpha} = \frac{2\pi i \mu}{h} G_{\alpha\alpha} \left(\frac{\partial \psi}{\partial x^\alpha}\right)^2 \frac{\partial \psi}{\partial x^i}. \tag{1.4}$$

$$G_{ij} = 0 \qquad i \neq j$$

while the equation of continuity becomes

$$\left(G \frac{\partial \psi}{\partial x^\alpha}\right)_\alpha = 0.$$

Writing the last of equations 1.4 in full, we obtain the relation

$$\frac{\partial}{\partial x^\alpha}\left(\mu\frac{\partial\psi}{\partial x^4}\right)\frac{\partial\psi}{\partial x^\alpha}+\mu\Gamma^4{}_{\alpha\beta}\frac{\partial\psi}{\partial x^\alpha}\frac{\partial\psi}{\partial x^\beta}=$$

$$\frac{\partial}{\partial x^4}\left(\mu\frac{\partial\psi}{\partial x^4}\right)\frac{\partial\psi}{\partial x^4}+\mu\Gamma^4{}_{\alpha\beta}\frac{\partial\psi}{\partial x^\alpha}\frac{\partial\psi}{\partial x^\beta}=\frac{2\pi i\mu}{h}G_{\alpha\alpha}\left(\frac{\partial\psi}{\partial x^\alpha}\right)^2\frac{\partial\psi}{\partial x^4},$$

an equation which can be further simplified since the g's are independent of x^4 in any periodic mechanical system for which the potential function is independent of the time.

From the definition of $\Gamma^4{}_{\alpha\beta}$ and the result that

$$\frac{\partial g_{ii}}{\partial x^4}=0 \qquad i=1\cdots 4$$

we have that $\Gamma^4{}_{\alpha\beta}=0 \qquad \alpha \text{ or } \beta \neq 4,$

so that the above equation can be written in the form

$$\left[\frac{\partial}{\partial x^4}\left(\mu\frac{\partial\psi}{\partial x^4}\right)+\mu\Gamma^4{}_{4\beta}\frac{\partial\psi}{\partial x^\beta}+\mu\Gamma^4{}_{\alpha 4}\frac{\partial\psi}{\partial x^\alpha}\right]\frac{\partial\psi}{\partial x^4}=$$

$$\frac{2\pi i\mu}{h}G_{\alpha\alpha}\left(\frac{\partial\psi}{\partial x^\alpha}\right)^2\frac{\partial\psi}{\partial x^4},$$

or since $$\frac{\partial\psi}{\partial x^4}\neq 0$$

$$\frac{\partial}{\partial x^4}\left(\mu\frac{\partial\psi}{\partial x^4}\right)+\mu\left\{\Gamma^4{}_{4\beta}\frac{\partial\psi}{\partial x^\beta}+\Gamma^4{}_{\alpha 4}\frac{\partial\psi}{\partial x^\alpha}\right\}=\frac{2\pi i\mu}{h}G_{\alpha\alpha}\left(\frac{\partial\psi}{\partial x^\alpha}\right)^2.$$

From the equation of continuity, we have that

$$\frac{\partial}{\partial x^4}\left(G\frac{\partial\psi}{\partial x^4}\right)+\Gamma^4{}_{4\beta}\frac{\partial\psi}{\partial x^\beta}=-\left\{\left(G\frac{\partial\psi}{\partial x^1}\right)_1+\left(G\frac{\partial\psi}{\partial x^2}\right)_2+\left(G\frac{\partial\psi}{\partial x^3}\right)_3\right\},$$

so that provided $G=k\mu$ where $k=$ constant, we arrive at the equation

$$\left(\mu\frac{\partial\psi}{\partial x^1}\right)_1+\left(\mu\frac{\partial\psi}{\partial x^2}\right)_2+\left(\mu\frac{\partial\psi}{\partial x^3}\right)_3-\Gamma^4{}_{\alpha 4}\left(\mu\frac{\partial\psi}{\partial x^\alpha}\right)+\frac{2\pi i\mu}{h}G_{\alpha\alpha}\left(\frac{\partial\psi}{\partial x^\alpha}\right)^2=0. \qquad (1.5)$$

The assumption that the density of electric charge, G, is a constant has no special significance since we are ignorant of the distribution of charge within an electron.

Limiting our discussion now to the symmetrical linear element, of which a familiar example is the Einstein space of the solar system, composed of a planet and a central body, the sun, where g_{44} is a function of x^1 only and $g_{11} = -\sqrt{-1}\, g_{44}^{-1}$ we have that

$$\left(\mu \frac{\partial \psi}{\partial x^1}\right)_1 = \frac{\partial}{\partial x^1}\left(\mu \frac{\partial \psi}{\partial x^1}\right) + \Gamma^1_{1\alpha}\left(\mu \frac{d\psi}{\partial x^\alpha}\right),$$

but $$\Gamma^1_{1\alpha} = g^{11}\,\Gamma_{1\alpha,\,1} = \frac{1}{2}\frac{\partial}{\partial x^\alpha}\log g_{44}$$

so that

$$\left(\mu \frac{\partial \psi}{\partial x^1}\right)_1 = \frac{\partial}{\partial x^1}\left(\mu \frac{\partial \psi}{\partial x^1}\right) + \frac{1}{2}\frac{\partial}{\partial x^1}\log g_{44}\left(\mu \frac{\partial \psi}{\partial x^1}\right).$$

The equation obtained by the elimination of $\frac{\partial^2 \psi}{\partial x^{42}}$ between the equation of continuity and the fourth equation of motion, then takes the form

$$\frac{\partial}{\partial x^1}\left(\mu \frac{\partial \psi}{\partial x^1}\right) + \left(\mu \frac{\partial \psi}{\partial x^2}\right)_2 + \left(\mu \frac{\partial \psi}{\partial x^3}\right)_3 + \frac{2\pi i \mu}{h} G_{\alpha\alpha}\left(\frac{\partial \psi}{\partial x^\alpha}\right)^2 = 0 \tag{1.6}$$

for $\Gamma^4_{14} = \frac{1}{2}\frac{\partial}{\partial x^1}\log g_{44}$ and $\Gamma^4_{i4} = 0 \qquad i = 2, 3 \text{ or } 4$

since $g_{44} = g_{44}$ (x^1 only).

Defining now a new function S, expressed in terms of μ and ψ by the equation

$$\frac{2\pi i \mu}{h}\frac{\partial \psi}{\partial x^i} = \frac{\partial \log S}{\partial x^i} \qquad i = 1 \cdots 4 \tag{1.7}$$

we can write 1.6 in the form

$$\frac{\partial^2 S}{\partial x^{12}} + \frac{\partial^2 S}{\partial x^{22}} + \frac{\partial^2 S}{\partial x^{32}} = \left\{\left(\frac{G_{11}}{\mu} - 1\right)\left(\frac{\partial S}{\partial x^1}\right)^2 + \left(\frac{G_{22}}{\mu} - 1\right)\left(\frac{\partial S}{\partial x^2}\right)^2 + \left(\frac{G_{33}}{\mu} - 1\right)\left(\frac{\partial S}{\partial x^3}\right)^2\right\}\frac{1}{S} + \frac{G_{44}}{\mu}\left(\frac{\partial S}{\partial x^4}\right)^2\frac{1}{S} + \Gamma^2_{2\alpha}\left(\frac{\partial S}{\partial x^\alpha}\right) + \Gamma^3_{3\alpha}\left(\frac{\partial S}{\partial x^\alpha}\right) = 0. \tag{1.8}$$

A solution can always be found for the function S defined by equation 1.7 since in the case where the variables are separable, which we are now considering,

$$\psi = \psi_1(x_1, x_2, x_3) + \psi_4(x^4)$$

so that if we set

$$S = S_1(x_1 x_2 x_3)\, S_4(x^4)$$

we have that

$$\frac{2\pi i\mu}{h}\frac{\partial \psi_n}{\partial x^i} = \frac{\partial \log S_n}{\partial x^i}, \qquad n = 1 \text{ or } 4 \qquad (1.9)$$

or

$$S = e^{\frac{2\pi i\mu}{h}(\psi + \text{constant})}.$$

If we now give to the G's and g's the special values

$$G_{11} = G_{22} = G_{33} = \mu \text{ and } G_{44} = \frac{\mu}{\nu^2}(E + V),$$

where E is a constant, V the potential energy of the mechanical system and g_{22} and g_{33} are constants, then equation 1.8 takes the form

$$\frac{\partial^2 S}{\partial x^{12}} + \frac{\partial^2 S}{\partial x^{22}} + \frac{\partial^2 S}{\partial x^{32}} + \frac{4\pi^2\mu^2}{h^2}(E + V)S = 0, \qquad (1.10)$$

provided $\psi_4 = \nu t$ $\nu = \text{constant}$

or S has the form

$$S = e^{\frac{2\pi i\mu}{h}(\psi_1 + \psi_4 + \text{constant})}.$$

Equation 1.10, which we have deduced from the one equation of motion and the equation of continuity by a transformation of the function ψ—the Hamiltonian Principal Function—in the case where the variables are separable, is identical with Schrödinger's equation in the new wave mechanics.

The condition imposed by Schrödinger in order that the solutions of equation 1.10 shall give solutions applicable to quantum problems is that the function S be single valued with respect to the spatial coördinates x^1, x^2 and x^3. This condition imposed on the function S leads to the following condition on ψ, namely,

$$\psi_1(x_0^1, x_0^2, x_0^3) = \psi_1(x_1^1, x_1^2, x_1^3) + \frac{nh}{\mu}.$$

n – an integer,

where the set of coördinates x_0^1, x_0^2, x_0^3 represent the same point in space as x_1^1, x_1^2, x_1^3.

Since, however,

$$Q^i = \frac{\partial \psi}{\partial x^i}$$

$$\psi_1(x^i) = \sum_{i=1}^{3} \int_{\alpha}^{x} Q^i \, dx^i + \text{constant},$$

and the above condition assumes the form

$$\sum_{i=1}^{3} \int_{x_0}^{x_1} \mu Q^i \, dx^i = nh \qquad (1.11)$$

the Einstein quantum condition that the value of the integral $\sum_{i=1}^{3} \int p_i dq^i$ over a complete period is an integral multiple of Planck's constant.

The theorem we have enunciated at the beginning of this discussion can now be expressed more precisely as follows:

Knowing any function ψ—the Hamiltonian Principal Function—which is found as the simultaneous solution of the equation of motion

$$\mu (Q^i)_\alpha Q^\alpha = \frac{2\pi i \mu^2}{h} Q^i \left\{ Q^{12} + Q^{22} + Q^{32} + \frac{(E+V)}{v^2} Q^{42} \right\} \qquad (1.12)$$

$$i = 1, 2, 3 \text{ or } 4$$

where the constant μ represents the mass and V the potential energy of the system and the equation of continuity

$$\rho (Q^\alpha)_\alpha = 0,$$

ρ being the constant charge density of the system, in any case where the variables are separable, *i.e.*,

$$Q^i = \frac{\partial \psi}{\partial x^i},$$

a solution S of Schrödinger's wave equation

$$\frac{\partial^2 S}{\partial x^{12}} + \frac{\partial^2 S}{\partial x^{22}} + \frac{\partial^2 S}{\partial x^{32}} + \frac{4\pi^2 \mu^2}{h^2} (E+V) S = 0,$$

which represents the same mechanical problem, is

$$S = e^{\frac{2\pi i \mu}{h} (\psi + \text{constant})},$$

the condition of uniqueness imposed by Schrödinger's on the function S being equivalent to the condition of Einstein's, namely

$$\sum_{i=1}^{3} \int \mu Q^i dx^i = nh \qquad n = \text{integer}$$

the integral being taken over a complete cycle.

As an example of these processes, we shall consider the equation of motion of a rotator with fixed axis.

The linear element, then, has the form

$$ds^2 = A d\phi^2 + g_{44} dt^2 \qquad A = \mu r^2$$

where ϕ is the angle of rotation and t the time, so that the equations of motion are

$$\mu \left(\frac{d\phi}{ds}\right)_2 \left(\frac{d\phi}{ds}\right) + \mu \left(\frac{d\phi}{ds}\right)_4 \left(\frac{dt}{ds}\right) = \frac{2\pi i \mu^2}{h} \left(\frac{d\phi}{ds}\right) \left\{ r^2 \left(\frac{d\phi}{ds}\right)^2 + \frac{E}{\nu^2} \left(\frac{dt}{ds}\right)^2 \right\}$$

since the system has no potential energy and

$$\mu \left(\frac{dt}{ds}\right)_2 \left(\frac{d\phi}{ds}\right) + \mu \left(\frac{dt}{ds}\right)_4 \left(\frac{dt}{ds}\right) = \frac{2\pi i \mu^2}{h} \left(\frac{dt}{ds}\right) \left\{ r^2 \left(\frac{d\phi}{ds}\right)^2 + \frac{E}{\nu^2} \left(\frac{dt}{ds}\right)^2 \right\}$$

which may be written in the form

$$\frac{\partial}{\partial \phi} \left(\frac{d\phi}{ds}\right) \left(\frac{d\phi}{ds}\right) + \frac{\partial}{\partial t} \left(\frac{d\phi}{ds}\right) \left(\frac{dt}{ds}\right) = \frac{2\pi i \mu}{h} \left(\frac{d\phi}{ds}\right) \left\{ r^2 \left(\frac{d\phi}{ds}\right)^2 + \frac{E}{\nu^2} \left(\frac{dt}{ds}\right)^2 \right\}$$

and

$$\frac{\partial}{\partial \phi} \left(\frac{dt}{ds}\right) \left(\frac{d\phi}{ds}\right) + \frac{\partial}{\partial t} \left(\frac{dt}{ds}\right) \left(\frac{dt}{ds}\right) = \frac{2\pi i \mu}{h} \left(\frac{dt}{ds}\right) \left\{ r^2 \left(\frac{d\phi}{ds}\right)^2 + \frac{E}{\nu^2} \left(\frac{dt}{ds}\right)^2 \right\}$$

since $g_{22} = A =$ constant and g_{44} is a function of r only.

If we now assume that $\frac{dt}{ds} = i\nu$ we have from the second equation that

$$r^2 \left(\frac{d\phi}{ds}\right)^2 = E$$

which satisfies the first equation and the equation of continuity

$$\rho \frac{\partial}{\partial \phi}\left(\frac{d\phi}{ds}\right) + \rho \frac{\partial}{\partial t}\left(\frac{dt}{ds}\right) = 0.$$

From the condition of periodicity, we have that

$$\int_0^{2\pi} \mu r \left(\frac{d\phi}{ds}\right) r d\phi = nh,$$

or

$$\mu \sqrt{E} = \frac{nh}{2\pi r},$$

or

$$E = \frac{n^2 h^2}{4\pi^2 \mu^2 r^2},$$

from which we obtain at once the well-known value for the energy ε of a rotator given by the quantum mechanics

$$\varepsilon = \frac{A}{2}\left(\frac{d\phi}{ds}\right)^2 = \frac{\mu E}{2} = \frac{n^2 h^2}{8\pi^2 A},$$

since $A = \mu r^2$.

From 1.9 we have that the appropriate solution of the corresponding wave equation

$$\frac{\partial^2 S}{\partial \phi^2} + \frac{4\pi^2 \mu^2}{h^2} r^2 E \,.\, S = 0$$

is

$$S = e^{\frac{2\pi i \mu}{h}\,(\sqrt{E}\,.\,r\phi + \text{constant})},$$

for

$$r^2\left(\frac{d\phi}{ds}\right)^2 = \left(\frac{\partial \psi}{r \partial \phi}\right)^2 = E,$$

or

$$\psi_2 = r\sqrt{E}\,.\,\phi,$$

of which the real part is given by

$$S = \cos \sqrt{\frac{4\pi^2 \mu^2 r^2 E}{h^2}} \cdot \phi,$$

the same solution as that given by Schrödinger.

From the above example, we see at once the reason for the validity of the Bohr relationship that the frequency of the emitted radiation is connected with the constant E by the equation

$$h\nu = \varepsilon_1 - \varepsilon_2 = \frac{\mu}{2}(E_1 - E_2),$$

for from the fourth equation of motion 1.2 (or better 1.12) we have that

$$\mu (Q^4)_\alpha Q^\alpha = \frac{2\pi i \mu^2}{h} Q^4 \left\{ Q^{1^2} + Q^{2^2} + Q^{3^2} + \frac{(E+V)}{\nu^2} Q^{4^2} \right\}.$$

Now

$$(Q^4)_\alpha Q^\alpha = Q^\alpha \left\{ \frac{\partial Q^4}{\partial x^\alpha} + \Gamma^4_{\alpha\beta} Q^\beta \right\}$$

$$= \frac{\partial}{\partial x^1} \log g_{44} \,.\, Q^1 Q^4,$$

since g_{44} is a function of x^1 only, and $Q^4 = \dfrac{\partial \psi_4}{\partial x^4} = \nu$ from equation immediately below 1.10; so that

$$\frac{\partial}{\partial x^1} \log g_{44} \,.\, Q^1 = \frac{2\pi i \mu}{h} \left\{ Q^{1^2} + Q^{2^2} + Q^{3^2} + (E+V) \right\}.$$

If we now take g_{44} = constant, which does not restrict the generality of the conclusions in the least, then we have the familiar energy equation

$$(Q^1)^2 + (Q^2)^2 + (Q^3)^2 + V = -E \qquad (1.13)$$

where V represents the potential energy and E the total energy of the sytem for unit mass.

With the completion of this example, we shall now turn our attention to the proof of the following theorem, concerning the relationship existing between Maxwell's equations, the equation of continuity and Schrödinger's wave equation.

THEOREM II.

Any solution of the dynamical problem, in which the variables are separable, associated with the motion of any distribution of electric charge which satisfies the equation of continuity for the case where the charge density is constant with respect to the time, *i.e.*, the equation

$$\frac{\partial}{\partial x}(\rho Q^1) + \frac{\partial}{\partial y}(\rho Q^2) + \frac{\partial}{\partial z}(\rho Q^3) = 0$$

where Q^1, Q^2, and Q^3 are the components of the velocity vector, and which also satisfies the law of the conservation of energy, *i.e.*, the equation

$$m\left\{Q^{1^2} + Q^{2^2} + Q^{3^2}\right\} + \phi = \text{constant}$$

where ϕ is the potential energy density of the system, is a solution of Schrödinger's wave equation.

Now the solution of any dynamical problem arising from the consideration of the motion of a distribution of electric charges must lead to such values for the velocity vector that the equation of continuity is satisfied, *i.e.*, the equation,

$$\frac{\partial}{\partial x}(\rho Q^1) + \frac{\partial}{\partial y}(\rho Q^2) + \frac{\partial}{\partial z}(\rho Q^3) = 0$$

provided the charge density is a constant with respect to the time throughout the distribution, an equation, however, which can be further simplified if the dynamical equations of motion of the charge are of such a form that the variables are separable, since then there exists a function ψ such that

$$\rho Q^i = \frac{\partial \psi}{\partial x^i}.$$

Substituting now into the equation of continuity the values of the Q's in terms of the derivatives of the principal function ψ, we obtain finally the equation

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = 0. \qquad (2.1)$$

In addition, if the motion of the electric charge is of such a nature that no radiant energy is emitted or absorbed during the motion of the charged body, then the energy density E of the system must be a constant, or expressed algebraically

$$\left(\frac{\partial \psi}{\partial x}\right)^2 + \left(\frac{\partial \psi}{\partial y}\right)^2 + \left(\frac{\partial \psi}{\partial z}\right)^2 + \frac{2\rho^2}{m} \cdot \phi = \frac{2\rho^2}{m} E, \qquad (2.2)$$

where ϕ is the potential energy density of the system, and m the mass density.

Moreover, the values of the function ψ found as a solution of Laplace's equation must be such that the derivatives of ψ with

respect to the x's is real, *i.e.*, the velocity components must be real.

If we now introduce the new function S defined in terms of ψ by the equation

$$\frac{\partial \psi}{\partial x^i} = \sqrt{-1}\,\frac{\partial}{\partial x^i}\log S \qquad \text{or } S = e^{-i\psi},$$

then the two equations limiting the motion of the distribution of charge become

$$\frac{\partial^2 S}{\partial x^2}+\frac{\partial^2 S}{\partial y^2}+\frac{\partial^2 S}{\partial z^2}-\frac{1}{S}\left[\left(\frac{\partial S}{\partial x}\right)^2+\left(\frac{\partial S}{\partial y}\right)^2+\left(\frac{\partial S}{\partial z}\right)^2\right]=0$$

and

$$-\left[\left(\frac{\partial S}{\partial x}\right)^2+\left(\frac{\partial S}{\partial y}\right)^2+\left(\frac{\partial S}{\partial z}\right)^2\right]=\frac{2\rho^2}{m}(E-\phi)S^2.$$

Substituting the value for the sum of the squares of the derivatives of S given by the second into the first of these equations, we obtain the equation of condition

$$\frac{\partial^2 S}{\partial x^2}+\frac{\partial^2 S}{\partial y^2}+\frac{\partial^2 S}{\partial z^2}+\frac{2\rho^2}{m}(E-\phi)S=0, \qquad (2.3)$$

Schrödinger's wave equation.

If, therefore, we know the principal function, ψ, found as the solution for any distribution of electrical charge, of (i) the equation of continuity and (ii) the equation which expresses mathematically that the energy density of the system remains constant, then a solution S of Schrödinger's wave equation for the same problem is found at once from the relationship that

$$S = e^{-\sqrt{-1}\,(\psi+\text{constant})}. \qquad (2.4)$$

The values of S defined in this manner are only valid along the totality of the trajectories traced out by the motion of the totality of the elementary volume charges $\rho\,dx\,dy\,dz$ composing the distribution of electric charge. However, since in an analogous manner to that used by Schrödinger we can assign the distribution of charge density throughout all space in such a manner that the total charge is finite and the major part of the charge is concentrated within an arbitrarily small volume, we can obtain a valuation for S which has a definite existence throughout all space.

The investigation of the solutions of 2.1 and 2.2 is also the same as that of the investigation of the solution for the electrical potential function throughout the region considered where the electric intensity R must have the value specified by the equation 2.2 throughout that region, *i.e.*,

$$R^2 = \left(\frac{\partial\psi}{\partial x}\right)^2 + \left(\frac{\partial\psi}{\partial y}\right)^2 + \left(\frac{\partial\psi}{\partial z}\right)^2 = \frac{2\rho^2}{m}(E - \phi)$$

where E is a constant and ϕ has a definite form. The restriction on the problem that the velocity components are real is equivalent to the restriction on the electrical problem that the components of the electric intensity be real.

From Green's theorem that

$$\iiint \psi \nabla^2\psi \, dx\, dy\, dz = \iiint R^2 \, dx\, dy\, dz + \iint \psi \frac{\partial\psi}{\partial n} \, dS = 0$$

where the volume integral is taken over the region considered, the charge density being everywhere zero within it, and the surface integral over the boundary of that region, and where $\frac{\partial}{\partial n}$ denotes differentiation along the outward normal, we have, since $\nabla^2\psi = 0$ and $\frac{\partial\psi}{\partial n} = \sigma$, σ = the surface density of charge on the boundary of the region considered, that

$$\iiint R^2 \, dx\, dy\, dz + \iint \psi\sigma \, dS = 0,$$

which shows that the conditional equation 2.2 is equivalent to knowing the distribution of electrical energy throughout the region considered, since from the above equation we can regard the energy distributed throughout the space, the energy density being R^2.

On account of the condition imposed on the function S found as the solution of Schrödinger's wave equation that it shall be finite, continuous and single valued throughout the region of the finite plane, the function ψ must satisfy the equation of condition that

$$\sum_{i=1}^{3} \int \frac{\partial\psi}{\partial x^i} \, dx^i = n/2\pi \qquad n = \text{integer}$$

where the integral is taken over any closed path, a condition which in this case can be replaced by the following, that

$$\int (Xdx + Ydy + Zdz) = n/2\pi \qquad (2.5)$$

since the electric intensity $X = -\frac{\partial \psi}{\partial x}$.

This condition may also be stated in the following form: The work done in moving a unit electric charge around any closed circuit in the space where the electric field is of such a form that it satisfies the two sets of differential equations expressed in mathematical form in equations 2.1 and 2.2 must be equal to an integer divided by 2π.

We can now state the following theorem, whose proof we have demonstrated above.

THEOREM III.

Any solution of the potential function V of Laplace's equation in the charge free space surrounding or contained within any distribution of electric charge which satisfies the two conditions, first, that the value of the square of the electric intensity R must be equal to the specific function $(E - \phi)$, *i.e.*,

$$R^2 = \left(\frac{\partial V}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial y}\right)^2 + \left(\frac{\partial V}{\partial z}\right)^2 = E - \phi$$

where E is a constant, and ϕ a definite function prescribed by the form of the distribution of electric charge density, and, secondly, that the work done in moving a unit around any closed path in this field is equal to an integral multiple of $\frac{1}{2\pi}$, *i.e.*,

$$\int Xdx + Ydy + Zdz = n/2\pi \qquad n = \text{integer}$$

defines a definite solution S of the corresponding problem in Schrödinger's wave mechanics by the equation that

$$S = e^{i(V + \text{constant})}.$$

INTEGRAPH SOLUTION OF DIFFERENTIAL EQUATIONS.

BY

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Both of the Massachusetts Institute of Technology.

ABSTRACT.

IN A previous article [1] a continuously recording integraph was described, by means of which differential equations, involving only one integration, could be solved. The present article describes a revision of this machine such that an equation involving two successive integrations, corresponding to practically any second-order total differential equation, with all terminal conditions included, can be solved. The need for a workable means of solving the differential equations involving empirical and discontinuous coefficients which occur repeatedly in electrical engineering and physics is recalled. In the machine described such solutions are effected by means of suitable interlinked integrating devices, the result being plotted continuously as a function of the independent variable. Tests and simple solutions show the over-all error to be approximately 1 or 2 per cent. The various sources of this error are discussed.

INTRODUCTION.

Formal mathematical methods have been and are invaluable in all branches of scientific work, and in problems where coefficients are constant, formal methods are often quite satisfactory. Many such problems in which this relationship exists actually, or is made to exist for the purpose of mathematical analysis by simplifying assumptions, can be solved this way. Where coefficients are not constant but are replaced by variable parameters, often represented only by experimental data, formal methods lose their applicability unless these parameters are approximated by empirical equations. Illustrative examples include vacuum tube circuits where the characteristics of vacuum tubes are involved; circuits containing iron where the saturation of the iron destroys the linear relationship between current and flux; circuits involving

[1] V. Bush, F. D. Gage, and H. R. Stewart, "A Continuous Integraph," JOUR. FRANK. INST., Jan., 1927, pp. 63–84.

rectifiers where the resistance of the rectifying element is a function of the current flowing through it. In these and numerous others, straightforward mathematical analysis to a final result is exceedingly laborious, if not impossible, because of the complexities introduced by trying to represent empirical relationships by mathematical expressions. Even in problems where linear relationships hold, the labor involved may be tremendous.

Where experimental data are involved, the natural and logical method of attack requires the use of these data in the form of curves or graphs as the only suitable method of representing it. Graphical methods constitute the logical attack both here and in problems where formal methods, though applicable, are impracticable.

By plotting experimental data, the nearest simple approach to the true representation of an experimentally determined function is obtained. Deviations of individual points, due to experimental error, from the true law of the function, which in general is continuous, are properly weighted in drawing the best representative curve. Such a curve then contains the best information available regarding this particular function. In some cases this function may follow a law simply expressed mathematically, in which case formal methods apply satisfactorily. If the function is double valued, discontinuous, or has other than very simple geometrical form, useful analytical representation is usually impossible with our present knowledge. The only alternative is the use of the curve itself as representing the function.

It was this line of reasoning which led to the construction of the original machine which could perform a single integration. Previous experience [2] with this machine, in the direct use of curves for mechanical solutions where only one integration is required, has demonstrated that this method has ample accuracy for engineering purposes and is otherwise distinctly satisfactory. Several unique problems have been solved on this machine. Its application, however, was limited in general to first-order equations.

First-order differential equations represent numerous phenomena in the fields of physics and electrical engineering, but a far

[2] Bush and Gould, "Temperature Distribution along an Emitting Filament," *Phys. Rev.*, Feb., 1927, pp. 337–345.

greater number of phenomena require for their mathematical description a differential equation of the second order. For example, the current in a circuit containing resistance, inductance, and capacity is represented as a function of time by a second-order equation. Problems in mechanics involving acceleration require a similar form of equation. It was once said that physics revolved about the second-order differential equation, and while recent developments have somewhat obscured this importance there is still much of physics thus described. It is therefore very desirable that a machine capable of handling a second-order differential equation of a form very nearly approaching the most general form be constructed. Of course, for systems of several degrees of freedom, it is desirable to be able to solve simultaneous equations of this type. The present machine deals with only a single equation, but may sometimes be used on simultaneous equations by the use of successive approximations. An extension of the principles involved will make possible the solution of simultaneous second-order equations, or of simple equations of higher order.

The present paper describes such a machine which, while not quite capable of handling the most general form of second-order differential equation, will solve readily a form which can be made to include essentially all second-order differential equations with one independent variable which are met in practice. Sample solutions are described and the method by which any particular equation can be solved by the machine is indicated. A discussion of practical operation, including the errors likely to arise, their magnitude, and methods of eliminating them, is included.

PRINCIPLE OF OPERATION.

The Original Machine.—A brief résumé of the operation of the original machine will be given to facilitate the understanding of its part in the new machine.

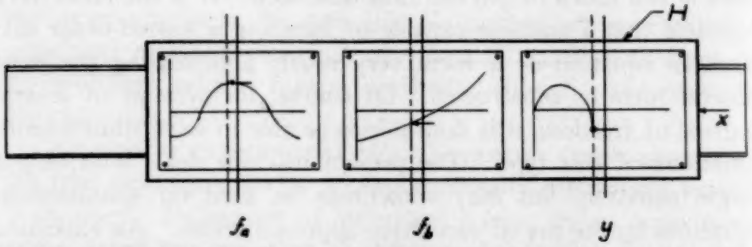
A platen H (Fig. 1) driven at uniform speed carries all of the curve sheets. These have the independent variable (designated as x) plotted along their abscissas which lie parallel to the axis of motion of the platen. Two indexes, say f_a and f_b, are free to move in a direction perpendicular to the x-axis. As the table moves, these pointers may be made to follow any curve

desired on the sheets. A third index y, operated automatically by the machine, always moves so that its displacement from the original position is given by

$$y = \int_m^x f_a f_b \, dx . \tag{1}$$

As the table moves, f_a and f_b are made to follow, respectively, the curves $f_a(x)$ and $f_b(x)$. The pointer y carries a pencil

FIG. 1.

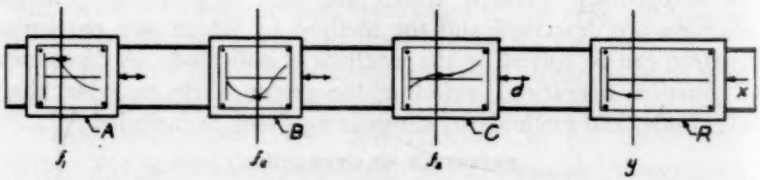


Schematic diagram of the original integraph for performing one integration. f_a and f_b are functions of x. The integral y of their product is plotted by the machine as a function of x.

recording the above integral continuously as a function of its upper limit x.

Present Machine.—On the new machine the original platen H is replaced by four independent platens, A, B, C, and R (Fig. 2), so that each index may have an individual platen whose motion

FIG. 2.



An extension of the machine shown in Fig. 1, such that first-order differential equations can be solved readily. f_1, f_a, and f_b independently may be functions of either y or x.

may be independent of that of the other platens. A further change is made so that the original factor f_b is now the sum of two functions f_1 and f_2 so that

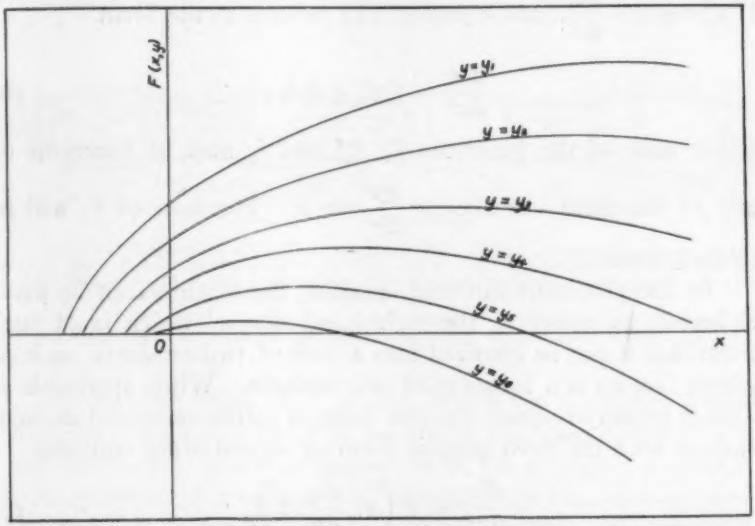
$$y = \int_m^x f_a (f_1 + f_2) \, dx . \tag{2}$$

Platen R on which the result index y records is always driven at uniform speed, its displacement from the initial position being

the value of the independent variable x. Each of the other platens, A, B, and C, may be driven so that its displacement from the initial position is equal to x, or equal to the result y of the integration. It will be seen that curves whose ordinates are functions of either x or y as abscissas may be used for any one of the three indexes to follow. Thus, each of the functions under the integral sign, f_a, f_1, and f_2, may be any function of x or y which can be represented by a graph.

The second stage of integration is put on in the following

FIG. 3.



Illustrative plot of a function of two variables. By the use of such plots when necessary, the complexity of the equations solvable by the integraph is greatly increased.

manner. To the result y of the first integration, the quantity given by the displacement of the index f_d is added (Fig. 3). This sum $(y + f_d)$ is then integrated to give the result

$$z = \int_l^x (y + f_d)\, dx\,. \tag{3}$$

From the previous equation for y it will be seen that

$$z = \int_l^x f_d\, dx + \int_l^x \int_m^x f_a\, (f_1 + f_2)\, dx\, dx\,. \tag{4}$$

The result z is plotted against the independent variable x on the platen R. The machine is so constructed that each of the platens

A, B, and C, may be made to move not only longitudinally according to either x or y, but also z. Therefore f_a, f_1, f_2, or f_d may be made functions of x, y, or z, by placing the proper curve on each platen and driving the platen so that its displacement is equal to the value of the corresponding variable. Equation (4) when expressed in differential equation form (obtained by two differentiations) is

$$\frac{d^2z}{dx^2} = f_a f_1 + f_a f_2 + \frac{d}{dx} f_d . \tag{5}$$

If $f_d = 0$, $y = \frac{dz}{dx}$, and equation (5) reduces to the form

$$\frac{d^2z}{dx^2} = f_a f_1 + f_a f_2 , \tag{6}$$

where each of the quantities f_a, f_1, and f_2 may be functions of any of the three variables x, $\frac{dz}{dx}$, or z. The uses of f_d will be treated below.

By the procedure outlined thus far, the machine can be made to handle an equation, the right-hand side of which is of such form that it can be resolved into a sum of two products, each of whose factors is a function of one variable. While applicable to a large group of equations, this form is rather restricted in comparison with the most general form of second-order equation,

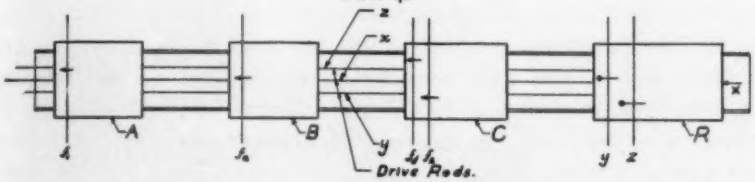
$$\frac{d^2z}{dx^2} = F\left(x, \frac{dz}{dx}, z\right), \tag{7}$$

containing on the right-hand side a general function of three variables. However, a much closer approach to this general case can be made. To show how this is done, let us consider a particular function $F(x, y)$ (Fig. 4) which is plotted against x as abscissa. Then $F(x, y)$ will have, for any particular value of x, an infinite number of values, depending on the value of y. The complete function can, however, be represented by plotting it against x for various definite values of y as a parameter ($y = y_1$, y_2, y_3, etc.). When such a family of curves is placed under one of the pointers, the corresponding platen moved according to x, and the particular value of y at each instant determined from the position of the y pencil at that instant, the setting of the pointer

which is following $F(x, y)$, is fixed. From this it will be seen that by thus interpolating as the machine proceeds any or all of the four independent functions f_a, f_1, f_2, and f_d can be functions of two variables, the result being a rather general type of second-order equation.

Since the machine integrates between definite limits, the solution obtained is not only *a* solution of the differential equation, but the complete solution of that equation for the given terminal or boundary conditions. Although neither equation (5) nor (6) becomes identical with (7) when the two-variable functions are used, it is very seldom in practice that a slight manipulation

FIG. 4.



Schematic diagram of the present integraph showing the arrangement of the various pointers (f_a, f_1, f_2, and f_d); the platens (A, B, C, and R); the x, y, and z-drive rods; and the recording pencils y and z. The displacement of R is always the independent variable x, while the displacement of A, B, and C may be independently x, y, or z.

of a particular equation will not reduce it to the form of (5) or (6).

The starting-point of x, y, and z, and the connected platens when the machine is put into operation may be chosen at will. Thus may be set, as will be shown below, the boundary conditions of the problem in question. By choosing various starting positions families of curves may be obtained corresponding to solutions of the equation for a series of values of the boundary conditions or the constants of integration.

DESCRIPTION OF MACHINE.

In the preceding section is made a statement of what equations the integraph will solve, without any mention of the method by which this result is accomplished mechanically. In this section are given descriptions of the electrical and mechanical details of the integraph.

The mechanism of the machine may be divided, for convenience in picturing it, into three parts, although in the actual machine these are very closely related. The three groups are:

First, the integrating devices themselves; second, the means by which the results of the integrations are conveniently recorded; third, the mechanisms by which the variables are introduced into the integrating devices. In addition to these necessary elements of the machine certain mechanical features used to improve the actual operation and accuracy will be described.

The integrating devices themselves will be considered first. The initial stage of integration is performed by a modified Thompson direct-current integrating watthour meter; the second stage by a Kelvin integrating wheel which is entirely mechanical. The first stage was described in the first paper, and its action may be summed up as follows: The speed of rotation of the moving element in a direct-current watthour meter is proportional to the instantaneous product of the currents in its potential and current coils. The total number of revolutions of the element is therefore proportional to the time integral of the product of these two currents. By means of potentiometer slide wires the potentials applied to these coils, and therefore the currents through them, may be continuously adjusted to any desired values. The construction is such that the potential applied is directly proportional to the ordinates of a curve which an index on the slide wire is made to follow. By this means the integral of the product of two functions can be obtained. If it is desired to make one of these factors the sum of two functions, the field coils may be so connected to two slide wires that the potential applied to the field winding is proportional to the algebraic sum of the two pointer displacements. By this means, then, the watthour meter may be made to give, instead of the product of two functions, the product of one function by the sum of two others. This is a simple extension of the original idea. This first stage of integration is used to evaluate the integral

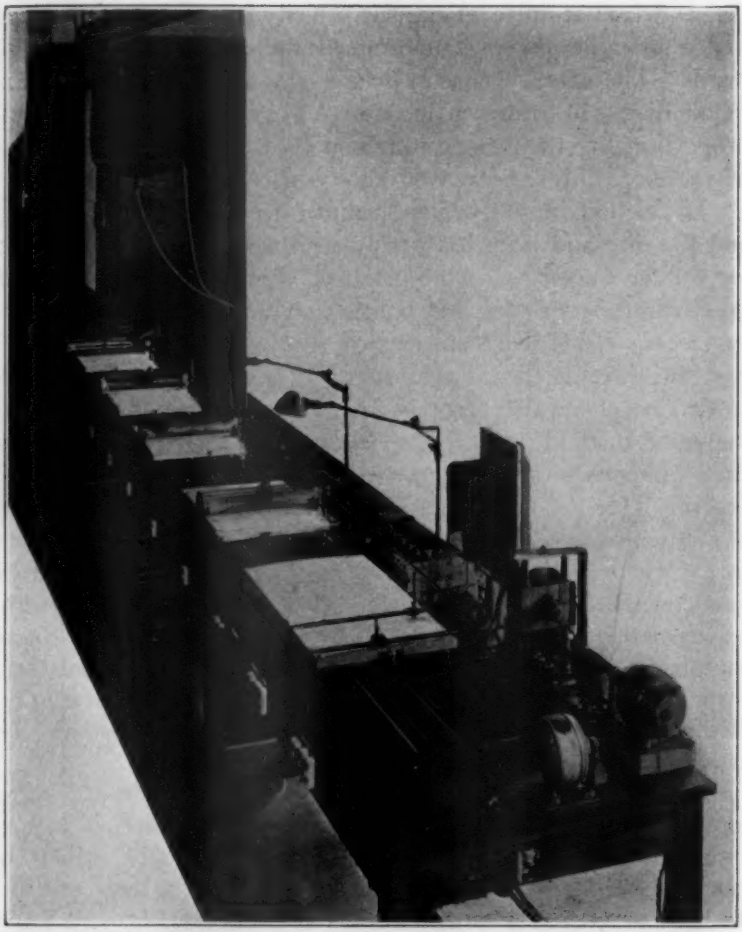
$$y = \int_m^x f_a (f_1 + f_2)\, dx . \qquad (2)$$

The second stage of integration, *i.e.*, the one which evaluates

$$z = \int_l^x (f_a + y)\, dx , \qquad (3)$$

employs the Kelvin integrating wheel. A visualization of this stage may be aided by thinking of it as identical in principle with

FIG. 5.



General view of the integraph. At intervals along the table are the platens which carry the plots used in solutions. At the right centre on a vertical panel is the modified Thompson watthour meter which performs the first integration. Below and to the right of this is the second-stage mechanical integrator.

From front to back the platens are R, C, B, A, and M, the latter serving for the multiplier described in the first paper. Slide wires for f_2, f_3, and f_1 are over platens C, B, and A, respectively. Across R are the y and z-recording screws with half nuts carrying pencils which record the results. The platens slide on ways made of 12″ channel section with one leg planed flat, the other planed to a V-section. Within the ways in the foreground from left to right are the y, x, and z-drive screws, driving the corresponding rods under the platens. The left and right motors in the foreground are, respectively, the x-drive and disc-drive synchronous motors. The cooling system condenser (an automobile radiator) with the water and steam connections to the slide wires is shown.

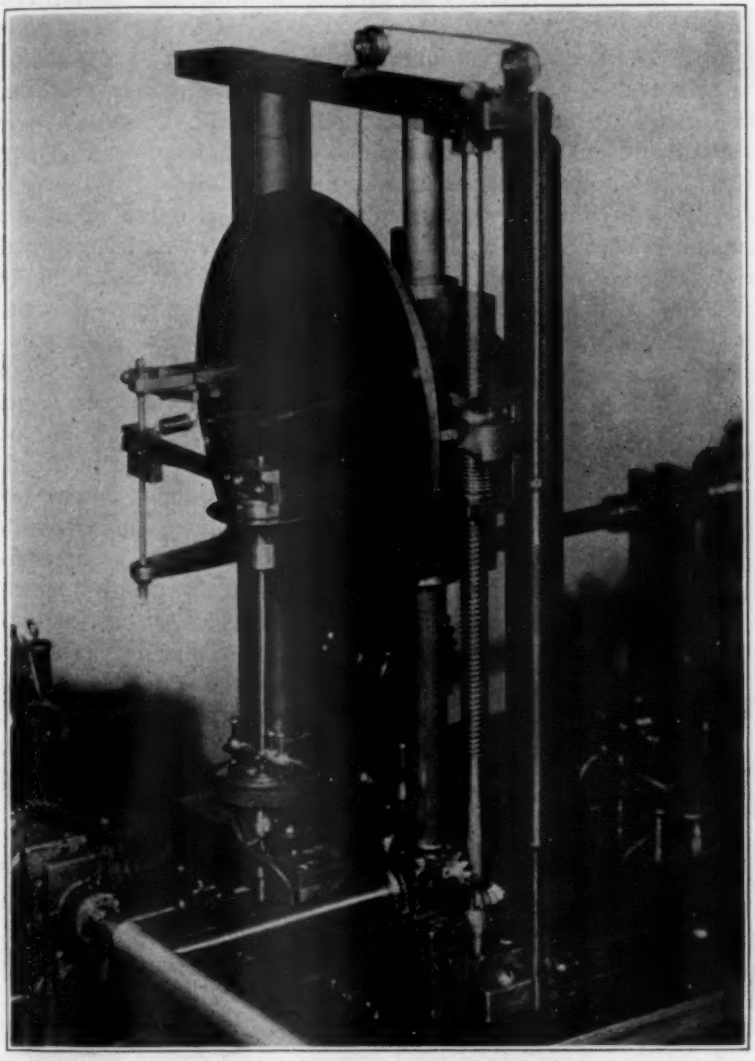
Under the left edge of the table are the various control switches.

a disc and wheel friction transmission where the disc is the driver and the wheel the driven member. In this integrating device the disc is driven at uniform angular velocity by a small synchronous motor operating on constant frequency alternating-current supply mains. The wheel, whose axis intersects at right angles the axis of the disc, will be driven at an angular velocity which is directly proportional to the distance of the point of contact from the axis of the disc. Figs. 6*a* and 6*b* show this device assembled.

If friction in the driven member be substantially removed and the disc and plate have truly polished surfaces this integrator will have no error. Both disc and wheel are made of hardened steel, carefully ground and polished, which gives nearly perfect surfaces. While the coefficient of friction between these two is low, the wheel is turned against practically zero retarding torque so that no observable error is found due to this cause. Exactly the same method is used for driving the recording mechanism and eliminating the friction torque on the wheel shaft as that described for the first stage in the first paper. The top bearing of the wheel shaft, which takes all the thrust produced by pressure of the wheel against the disc, is a small precision ball-bearing introducing a negligible torque. The bottom bearing, which serves merely as a guide, is driven by the servo-motor, and rotates back and forth with respect to the shaft resting in it, and therefore introduces a slight torque due to bearing friction, first in one direction and then the other in approximately equal amounts. The net result is an essentially frictionless bearing.

As can be seen from Fig. 6*a* the displacement of the point of contact of wheel and disc from the axis of the disc is secured by moving, in a vertical direction, the carriage on which are mounted the disc bearings. Rotational drive to the disc is secured through a vertical splined shaft driving the disc through a pair of hardened steel spiral gears. This, with the synchronous motor drive through change gears, is shown in Fig. 6*b*. It will be seen that the ordinate which is integrated is the vertical displacement of the disc carriage above or below the position at which the point of contact between the wheel and disc lies on the axis of the disc. This vertical displacement is secured by a vertical Acme screw on which runs a self-aligning nut fastened to the carriage. Lost motion at this point would result in an error which in many

FIG. 6a



This view shows the disc and integrating wheel of the mechanical integrator. The carriage carrying the disc slides vertically on ¾" diameter steel ways, its vertical position being adjusted by the Acme screw on which runs a self-aligning nut. To equalize the thrust on the screw, the carriage is counterbalanced.

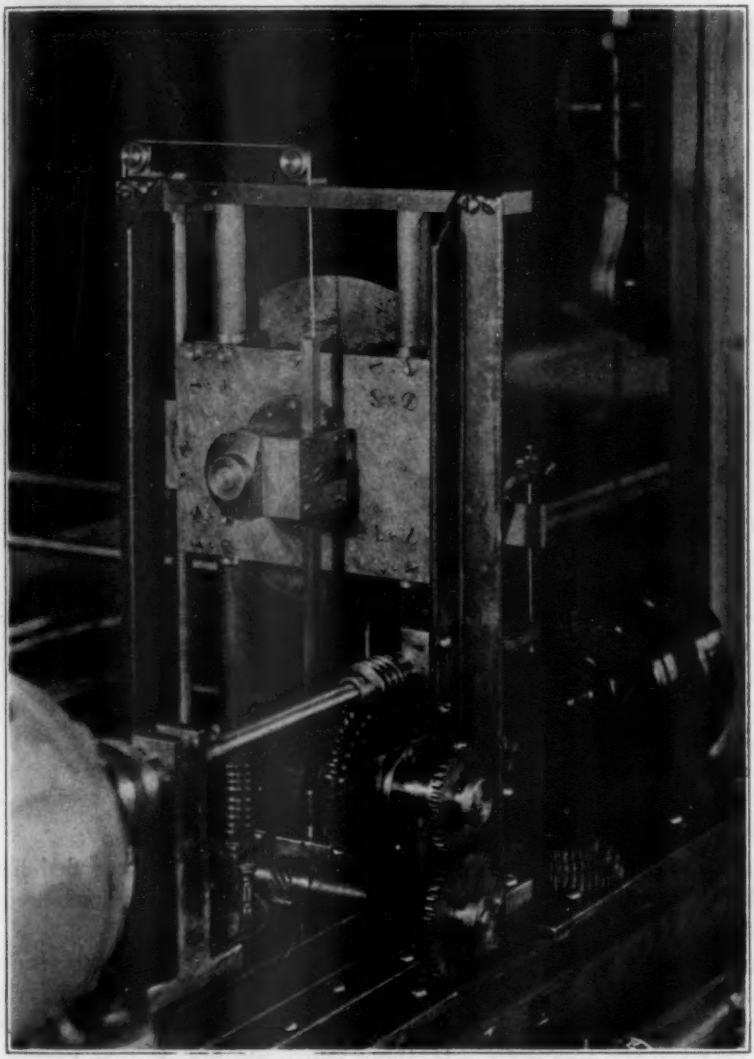
types of problems would become cumulative, and hence an accurate screw and nut with practically no backlash are necessary.

The foregoing describes the mechanical integrator itself. The function which is thus integrated is the sum of the two functions as previously described in connection with equation (3). One of these functions is the result y obtained from the first integration, the second an independent function f_d. The function f_d is inserted into the machine by manually turning a screw carrying a half nut with an index which is made to follow the function f_d. (Fig. 7.) Both y and f_d are therefore represented by the number of revolutions which their controlling screws have turned from arbitrary zero points. To add these functions and to apply their sum to the second stage of integration, it suffices that the two screws of f_d and y be connected to two elements of a mechanical differential gear, the third element being geared directly to the screw which gives vertical motion to the integrator disc carriage.

The second portion of the mechanism, *i.e.*, that by means of which the results are read and recorded, will next be considered. As has been indicated, the results of the integrations are recorded as ordinates on coördinate paper fastened to the platen R, with any particular line parallel to the motion of the table as an arbitrary zero. The magnitude of the ordinate at any point is equal, on a scale determined by calibration, to the value of the integral with the given value of the independent variable as the upper limit. Now the integrating devices give the value of the integral as the total number of revolutions which a given shaft has turned. Correlation between these two is obtained by placing a half nut, to which is fastened a pencil, on a thread cut in the recording shaft which is being driven indirectly by the integrator shaft. The translation of the pencil is thus directly proportional to the number of revolutions which the recording shaft has turned and therefore proportional to the value of the integral. Both y and z are recorded in this manner on the same sheet of graph paper, each having its own zero of abscissas and the same zero or different zeros of ordinates as desired.

It was stated that each recording shaft is moved indirectly by the integrator shaft whose total angular displacement, amounting to many revolutions, is the value of the integral to be recorded. It is essential that these integrator shafts—in the first stage, the watthour meter rotor; in the second, the wheel shaft—be free

FIG. 6*b*.



The rear view of the mechanical integrator shows the splined shaft and spiral gears used to drive the disc. The worm drive from a ¼-h.p. synchronous motor, and the change gears, by means of which scales are changed, are seen in the lower part of the picture.

from all friction and load torque, and hence they cannot directly furnish energy to drive the recording shafts. A servo-motor follower mechanism is therefore used to drive each recording shaft, and this not only reduces the necessary energy output of the integrator shafts to a negligible value, but, as mentioned above, practically eliminates the effect of bearing friction on these shafts at the same time. This mechanism is really the key to the success of the machine from the practical mechanical point of view.

Thus far the process by which certain functions designated

FIG. 7.

A potentiometer slide wire mounted in position across a platen is shown. In addition to the slide wire this platen (*C*) serves also the f_4 index, carried on the half nut running on the manually driven screw.

by f_1, f_2, f_3, and f_4 are integrated and the results recorded has been described. How these indexes are made to be functions of the desired variables will now be considered.

In the discussion of equation (4) it was stated that the independent variable x, the result y of the first integration, or the result z of the second integration, could be used as variables in any of these functions. If these functions are of one variable only, this variable is taken as the abscissa of a plot whose ordinates are the value of the function. This plot is fastened to the platen which moves under the index whose displacement is to be

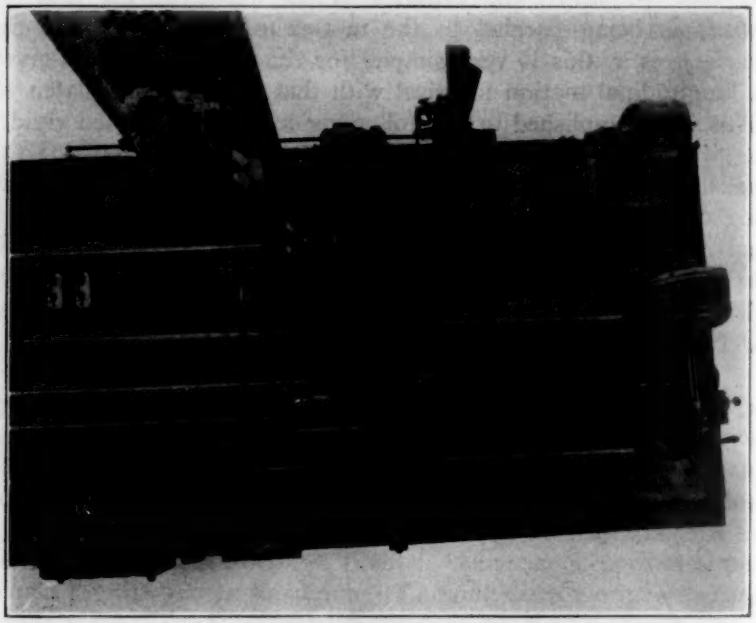
made equal to the given function. The index is free to move only in a transverse direction across the platen and is operated manually to follow the plot of the function, as the platen is driven along the machine bed under the index. (Fig. 7.) The longitudinal displacement of the platen from the position where the index rests on the zero of abscissas, is made equal to the value of the variable of which the ordinate is the function, the axis of abscissas being parallel to the motion of the platen. If the variable is x, this is very simple, for the platen is merely given a longitudinal motion identical with that of the result platen R. This is accomplished in the following manner: Fastened rigidly to platen R is a steel tube which runs the entire length of the machine bed under all the platens. On each platen is a slide bearing through which this tube passes. If it is desired to make a particular platen move as a function of x, a set screw in this bearing is tightened against the drive tube so that the table is driven directly by the longitudinal motion of the tube. Any or all of the platens A, B, and C can be thus driven. (Fig. 5.)

If a particular function has for its independent variable not x, but y or z, the process is similar. Geared to the recording screw y is a lead screw placed on the bed of the machine under platen R with its axis longitudinal. On this screw runs a half nut which is fastened to a tube similar to the x-drive tube and which likewise runs under all the platens. For a function of y the screw on the particular platen is tightened on the y-drive tube instead of the x-drive tube. The mechanism for giving a platen a displacement equal to z is identical with that used for the y drive. The net result, as stated above, is that any platen may be made to move so that its displacement is equal to any one of the three variables x, y, or z. (Fig. 8.) Some of the mechanisms which are used to facilitate the practical operation of the machine, and certain devices which are necessary to eliminate errors, will now be considered.

In order to understand the use of several of these devices an examination of the causes of certain errors will be made. If one integration is being performed, say, that using the watthour meter, the principal requirements for accuracy are that the watthour meter speed be strictly proportional to the product of the two ordinates being integrated, and that its position as a function of time be accurately transferred to the record. If for any reason

the displacement of the recording pencil lags a small amount behind the position which it would have were there no lag errors in the instrument, there is a small error introduced. By lag error is meant the difference between the actual position of the recording pencil and that which it would have were the machine perfect.

FIG. 8.



A bird's eye view of the integrating, recording, and coupling mechanisms. At the right centre is the x-drive synchronous motor and change gears, driving the central screw. Crossing the ways are two screws, the left and right being the y and z-recording screws, respectively, each geared to its own longitudinal drive screw within the channel iron ways. The y and z-drive rods with the clamping nuts by means of which they are coupled to the drive screws are clearly shown. Normally platen R runs over these screws.

At the top centre is the z-servo-motor. At its right is the mechanical integrator, with its synchronous drive motor at the extreme upper right. Near it are the servo-controlling relays. The watthour meter integrator with its servo-mechanism and relays is at the upper left.

Below and slightly to the left of the z-servo-motor is the planetary differential gear which adds f_d and y.

This lag error usually causes no great difficulty when it is desired merely to integrate a function. If, however, the result of this integration is integrated again and the result of this second integration used as a variable, as is done when the machine is used to solve second-order equations, the lag error may become cumulative and therefore serious. It thus becomes necessary either to eliminate it or to compensate for it.

Lag is introduced principally in three ways. First, any backlash in the gearing produces a lost motion giving this result. Second, the servo-mechanism as previously described introduces an error of this nature in the following manner. Suppose the watthour meter rotor is turning at a given speed and the relay operating the servo-mechanism is in such a position that the servo-motor is being accelerated. Opposing its accelerating torque are the retarding torques due to friction and inertia of the moving parts. When the servo-motor has driven the follower mechanism to a position which is in advance of the position about which it oscillates, the relay reverses and this follower mechanism is retarded by the torque of the motor and friction torque of the mechanism, these retarding torques opposing the inertia torque. If the servo-motor torque is approximately constant in both directions, it will be seen that the average position of the follower mechanism will be somewhat behind its ideal position since friction causes the mechanism to slow down much more rapidly than it speeds up. The lag due to this cause is approximately proportional to the speed. The third source of lag is inherent in the watthour meter. The desired integral would be given exactly by a watthour meter whose rotor had no inertia, and on which no retarding torque other than that of the drag magnets, which is proportional to the speed, acted. In the actual watthour meter, the torque representing the function being integrated is opposed by an additional torque, that necessary to accelerate the rotor. Although the *speed* differs by a negligible amount from the ideal speed at any instant, the disc never will have turned through as great an angle as it should have turned through except at zero speed; *i.e.*, some of the energy put into the rotor is converted into kinetic energy of rotation which, had it been dissipated against torque produced by the drag magnets, would have caused the disc to be at a position slightly in advance of that which it actually occupies. When the disc is retarded from some particular speed to a standstill, this kinetic energy is dissipated against the drag magnet torque so that the final stopping position of the disc will coincide with the ideal position. It can be shown readily that this inertia lag angle, making the justifiable assumption that the friction torque is negligible, depends only on the instantaneous speed of the rotor, and is directly proportional to it.

The first of these lag errors, that due to backlash, can be

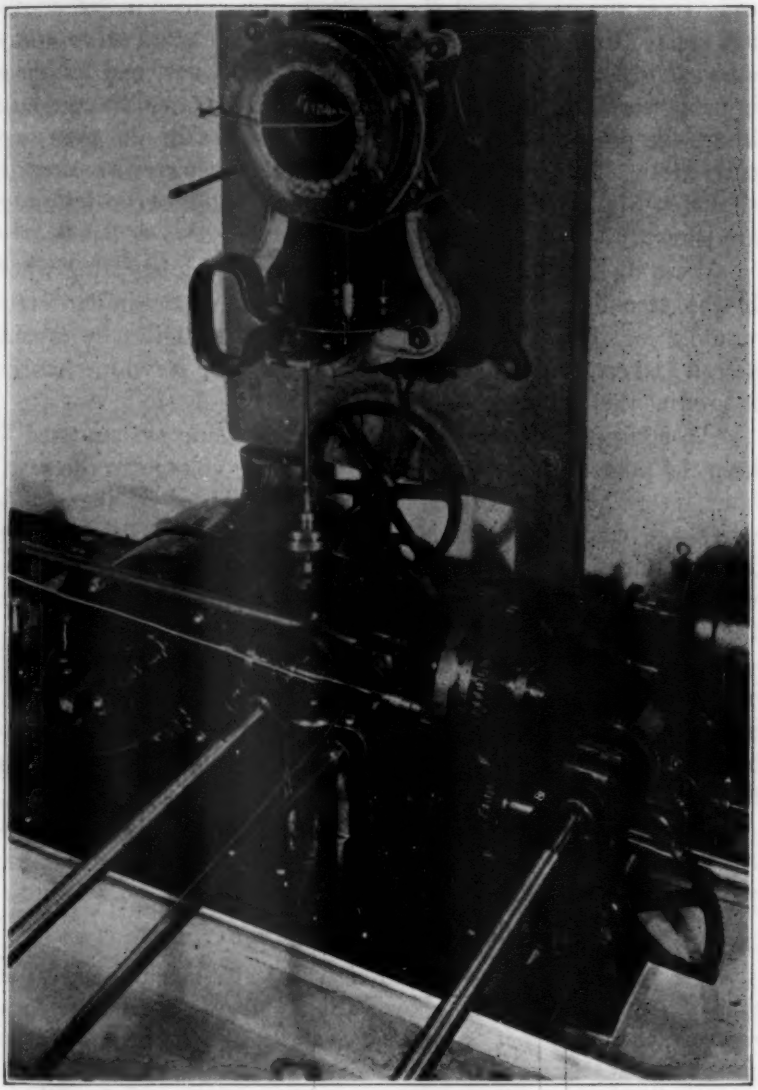
eliminated sufficiently well by reasonable care in design and construction.

The second or servo-mechanism lag can be reduced to a negligible quantity. This is done by using two servo-motors instead of one, these two adding their revolutions in a differential gear to give the resultant motion of the follower mechanism. The error in the original device was introduced by causing the relay-controlled servo-motor to carry more load in one direction than in the other. If this oscillating motor is made to oscillate about a stationary position, instead of a rotating reference, this error is eliminated. The second or main servo-motor does just this; that is, it supplies the necessary constant component of rotation. Suppose the watthour meter to be rotating at uniform speed with the main servo-motor driving the follower at exactly this speed, the secondary or control motor merely oscillating about a mean position. Let the watthour meter speed change. Nothing causes the speed of the main motor to change, hence the control motor starts making unequal oscillations resulting in an average speed such that when added to the speed of the main motor it gives the speed of the watthour meter. Now, if this net rotation of the control motor is used to alter the voltage applied to the main motor armature in such a way as to bring its speed to that of the new watthour meter speed, a stable condition results where the control motor again oscillates about a stationary position. This is accomplished by causing the control motor to drive also a potentiometer from which is derived the voltage applied to the armature of the main servo-motor. While this device sounds slightly complicated, it is entirely automatic and works very satisfactorily.

The third error, due to inherent inertia lag in the watthour meter disc, is compensated for as follows. Instead of the follower mechanism giving to the machine the exact position of the watthour meter disc at each instant, which lags the position it should have, it gives this position plus an angular displacement proportional to the speed at which the disc is then turning. This correction is of such magnitude that it causes the servo-mechanism to deliver the position which an ideal inertialess watthour meter would have, thus completely and effectively compensating for this error.

The necessary angular displacement is introduced through a

FIG. 9.



Watthour meter follower mechanism. At the top is the modified Thompson watthour meter, below which on the small vertical shaft is the planetary differential gear used to introduce the correction for the watthour meter inertia lag. At the left is the main y-servo-motor, the control motor at its right being largely obscured by gearing. The potentiometer operated by the control motor governing the speed of the main motor is under the table. A portion o the z-servo-motor is seen at the extreme right.

small differential gear, the displacement being obtained mechanically from the position of the potentiometer controlling the voltage applied to the main servo-motor. Since the main servo-motor speed is identical with that of the watthour meter, and its speed is proportional to its applied armature voltage, this voltage, and hence the potentiometer position, is a measure of the necessary correction. This mechanism, including the two servo-motors, is shown in Fig. 9. In the other pictures only single-servo followers are shown.

Were slippage to occur between the disc and integrating wheel in the second stage of integration, a somewhat similar error would arise. This factor has been rendered negligible by careful design and construction, and hence only a simple servo-mechanism is used at this point.

An additional detail of interest is the cooling system used to keep the potentiometer slide wires supplying voltage to the watthour meter at a reasonable temperature. Three of these slide wires are employed, each dissipating about one-half kilowatt. This requires auxiliary cooling which is provided by a closed water system, generating steam which is condensed in a radiator. Fig. 5 shows the layout of this cooling system.

Electrical limit stops, tripping circuit breakers, are used to prevent jamming due to overtravel of any part of the mechanism. Switches are arranged so that the servo-mechanisms may be independently put into operation, and the balance of the device then started by closing a single switch.

Having considered the salient points of the design and construction of the integraph, the process by which equations are solved on it will be taken up next.

APPLICATION TO SOLUTION OF EQUATIONS.

The general form of equation which the machine solves is that of (3) or (4) given above. Since all of the functions are represented by plots it is necessary, as in all graphical work, to choose the scales on which each of the variables is represented. These scales are interrelated, and certain of them having been arbitrarily selected, the remainder are determined. The choice of the independent scales, and the determination of the dependent scales is an important step in a given solution, and the relations

on which this scale determination is based will be considered before describing specific examples.

In performing any solution there are two distinct sets of variables involved which must be clearly distinguished. The machine of itself deals with linear distances and time alone, and these comprise the first set of variables. The other set is that involved in the equation as presented for solution. By proper correlation of these two sets of variables, the machine is made to solve the given equation. The equations actually solved by the machine will be discussed first.

Let equation (2) be rewritten as

$$y = b\int_m^x f_a\,(f_1 + f_2)\,dx \tag{8}$$

and (3) as

$$z = \frac{a}{b}\int_i^x (f_d + y)\,dx\,, \tag{9}$$

or

$$z = \frac{a}{b}\int_i^x f_d\,dx + a\int_i^x\int_m^x f_a\,(f_1 + f_2)\,dx\;\;dx\,, \tag{10}$$

where a and b are constants. If all the variables are expressed in some convenient unit of length, say centimetres, a and b can be evaluated by one simple run and will remain fixed thereafter.

Table I gives the values for a and b when all the variables are expressed in centimetres, and the range of each of the variables.

TABLE I.

Variable.	Range.	Variable.	Range.
x	0 to 60 cm.	f_2	−20 to +20 cm.
y	0 to 40 cm.	f_d	−20 to +20 cm.
z	0 to 40 cm.		
f_a	−20 to +20 cm.	a	0.000678
f_1	−20 to +20 cm.	b	0.00672

These values of a and b are for given standard gear ratios on the machine. To increase the range and flexibility of the machine, three sets of change gears are employed, and to describe completely the possibilities of scale variation, three additional

constants are used, each representing the ratio of one of these change gear sets. The scale to which y is recorded and introduced into the second stage of integration is proportional to the ratio $\beta = \frac{\text{driver teeth}}{\text{driven teeth}}$ in the y change gear group. The scale of z is similarly determined by a double gear set whose ratio is α. To allow for the different possible speeds of the result platen R, along which x is measured, the factor n is introduced. With these new constants included, (10) becomes

$$z = \frac{n\alpha a}{b}\int_i^x f_d\,dx + n^2\alpha\beta a\int_i^x\int_m^x f_a(f_1+f_2)\,dx\;dx, \quad (11)$$

which is the complete expression for the equation actually solved on the machine, all distances being expressed in centimetres. In (11) the values given in Table I apply with the addition of those in Table II.

TABLE II.

$\alpha = \frac{440}{504} \times \frac{\text{Driver teeth } A}{\text{Driven teeth } A} \times \frac{\text{Driver teeth } B}{\text{Driven teeth } B}$ on z change gears.

$\beta = \frac{\text{Driver teeth}}{\text{Driven teeth}}$ on y change gears.

$n = \begin{cases} 1 \text{ for high-speed } x \text{ drive.} \\ 2 \text{ for low-speed } x \text{ drive.} \end{cases}$

Sum of A teeth = 96.

Sum of B teeth = 80.

$\beta = \frac{1}{10}, \frac{1}{3}, \frac{1}{2}$, 1, 2, 3, or 10.

In general there is no restriction on the combination of ratios except the choice of suitable scales, discussed below.

Equation (11) is in reality a differential equation, though written in integral form. Since differential equations are customarily written, and therefore more familiar, in differential form, this form will be used in the remainder of the paper. By two differentiations (11) becomes

$$\frac{d^2z}{dx^2} = \frac{n\alpha a}{b}\frac{d}{dx}f_d + n^2\alpha\beta a f_a(f_1+f_2), \quad (12)$$

which is identical with (11) except that the terminal conditions are omitted. For any particular solution, each of the variables in (12) will represent, to some scale, a variable in the problem to be solved.

The process by which these scales are selected probably is best described by giving a simple illustration which will demonstrate the principles involved. Consider, for example, the equation of an electric circuit containing resistance, inductance, and capacitance, and the transient caused by discharging the condenser into the circuit.

$$L\frac{di}{dt} + ri + \frac{1}{C}\int idt = 0. \tag{13}$$

Differentiating once, transposing and dividing by L,

$$\frac{d^2i}{dt^2} = -\frac{r}{L}\frac{di}{dt} - \frac{1}{LC}i. \tag{14}$$

(14) is similar in form to (12) and can be solved on the machine in the following manner:

Let

$$\begin{aligned} z &= k_z i & f_1 &= -\frac{k_1}{LC} \\ x &= k_x t & f_2 &= 0 \\ f_a &= k_a i & f_d &= -k_d\frac{r}{L}i. \end{aligned} \tag{15}$$

These values in (15) will now be substituted in (12) after the necessary transformations.

$$\frac{dz}{dx} = \frac{d(k_z i)}{d(k_x t)} = \frac{k_z\dfrac{di}{du}}{k_x\dfrac{dt}{du}} = \frac{k_z}{k_x}\frac{di}{dt},$$

where u is any variable.

$$\begin{aligned} \frac{d^2z}{dx^2} &= \frac{d}{dx}\left(\frac{k_z}{k_x}\frac{di}{dt}\right) = \frac{d}{dt}\left(\frac{k_z}{k_x}\frac{di}{dt}\right)\frac{dt}{dx} \\ &= \frac{k_z}{k_x}\frac{d^2i}{dt^2}\frac{dt}{dx} = \frac{k_z}{k_x^2}\frac{d^2i}{dt^2} \end{aligned} \tag{16}$$

$$\begin{aligned} \frac{d}{dx}f_d = \frac{d}{dx}\left(-k_d\frac{r}{L}i\right) &= -k_d\frac{r}{L}\frac{di}{dx} = -k_d\frac{r}{L}\frac{di}{dt}\frac{dt}{dx} \\ &= -\frac{k_d}{k_x}\frac{r}{L}\frac{di}{dt}. \end{aligned} \tag{17}$$

Substituting (16), (17), and (15) in (12)

$$\frac{k_z}{k_x^2}\frac{d^2i}{dt^2} = -\frac{n\alpha a k_d}{bk_x}\frac{r}{L}\frac{di}{dt} - \frac{n^2\alpha\beta a k_a k_1}{LC}i. \qquad (18)$$

Dividing by $\frac{k_z}{k_x^2}$

$$\frac{d^2i}{dt^2} = -\frac{n\alpha a k_x k_d}{bk_z}\left(\frac{r}{L}\frac{di}{dt}\right) - \frac{n^2\alpha\beta a k_a k_1 k_x^2}{k_z}\frac{i}{LC}. \qquad (19)$$

When compared with (14) it will be seen that (19) is identical with (14) provided the following relations exist:

$$\frac{n\alpha a k_x k_d}{bk_z} = 1, \qquad (20)$$

$$\frac{n^2\alpha\beta a k_1 k_a k_x^2}{k_z} = 1. \qquad (21)$$

To carry through a definite solution, let

$L = 0.500$ henry
$C = 40.0$ mfd.
$r = 20.0$ ohms

and the initial voltage on the condenser be 200 volts. Since the analytical solution of this problem is very easy, portions of the solution will be made so that the final scales can be chosen immediately without a trial run using preliminary scales to determine the approximate solution.

If $r^2C < 4L$ the circuit will be oscillatory

$$r^2C = \frac{20^2 \times 40}{10^6} = 0.016$$

$$4L = 4 \times 0.5 = 2.0.$$

This shows the damping to be small and

$$\omega = \frac{1}{\sqrt{LC}} \text{ very closely}$$

$$= \frac{10^3}{\sqrt{0.5 \times 40}} = 223.8 \text{ radians/sec.}$$

$$f = \frac{\omega}{2\pi} = \frac{223.8}{2\pi} = 35.6 \text{ cycles/sec.}$$

$$T = 0.0281 \text{ sec.}$$

If there were no resistance, the maximum value of i would be such that

$$\frac{Li^2}{2} = \frac{Ce^2}{2}$$

$$i = e\sqrt{\frac{C}{L}} = \frac{200}{10^3}\sqrt{\frac{40}{0.5}} = 1.790 \text{ amperes.}$$

A convenient scale for i is 10 cm. per ampere, giving a maximum displacement of 17.9 cm. on both the positive and negative sides of the ordinate of z selected for zero. Suppose it is desired to solve one cycle in one traverse of the platen. Then 0.0281 sec. should require slightly less than 60 cm. and a scale of 2000 cm. per sec. is convenient. Having chosen these scales it is possible to evaluate nak_d from (20) and

$$nak_d = \frac{bk_z}{ak_x} = \frac{0.00672 \times 10}{0.000678 \times 2000} = 0.0496.$$

Now k_d is the scale of the quantity $\frac{ri}{L}$ or $\frac{\text{volts}}{L}$, which will have a maximum of $\frac{200}{0.5}$ or 400 and should be between 10 and 20 cm. on the plots for convenience. A scale of about 0.025 cm. per $\frac{\text{volts}}{L}$ is satisfactory and if $a = 1$ and $n = 2$

$$k_d = 0.0248 \text{ cm. per } \frac{\text{volt}}{L}.$$

From (21) the product $\beta k_1 k_a$ can now be determined

$$\beta k_1 k_a = \frac{k_z}{n^2 a a k_x^2} = \frac{10}{2^2 \times 1 \times 0.000678 \times 2^2 \times 10^6} = \frac{9.22}{10^4}.$$

Now

$$f_1 = -\frac{k_1}{LC} \text{ and } f_1 k_a = -\frac{k_1 k_a}{CL}$$

$$f_1 k_a = -\frac{9.22}{10^4 \beta LC} = -\frac{9.22 \times 10^6}{10^4 \beta 0.5 \times 40} = -\frac{46.1}{\beta}.$$

Taking $\beta = 1$ and $k_a = 5$ cm. per ampere will make $f_1 = -9.22$ cm., which is satisfactory.

Having selected all the scales, the problem may now be set up

on the integraph and solved. f_1 is given a fixed displacement of -9.22 cm. The displacement of f_a is to be $k_a i$, hence table B (Fig. 3) must be coupled to the z-drive rod whose displacement is always $k_z i$. If the index is made to follow a straight line on B whose slope is $\frac{k_a}{k_z}$, the displacement of f_a will be $\frac{k_a}{k_z} k_z i - k_a i$, which is the desired result. f_d is also to be a function of i, that is, $-k_d \frac{r}{L} i$. This is accomplished by making the f_d index follow a line plotted on table C, and coupling C to the z-drive rod. In this case the desired slope is $-\frac{k_d}{k_z}\frac{r}{L}$, since $-\frac{k_d r}{k_z L} \times k_z i = -k_d \frac{r}{L} i$. That is, the displacement of f_d is the longitudinal displacement of C multiplied by the slope of the line followed. Now the initial conditions are introduced which, in this case, are that at $t = 0$, $i = 0$, and the condenser voltage $V_c = 200$ v. Since $z = k_z i$ and $f_a = k_a i$ it follows that z must be on zero and that tables B and C must be clamped on the z-drive rod so that f_a and f_d are zero at the same time. It will be noted that f_d is the ir drop to the numerical scale $\frac{k_d}{L}$. βy is added directly to f_d at a one-to-one ratio, hence has the same dimensions as f_d, *i.e.*, $\frac{\text{volts}}{L}$. It will be seen that the voltage represented by the displacement βy is that on the condenser. Initially βy should have a displacement

$$k_d \frac{V_c}{L} = \frac{0.0248 \times 200}{0.5} = 9.92 \text{ cm.}$$

After this displacement is given to the y pencil, the starting switch can be closed at will. Two operators follow the curves on C and B with indexes f_d and f_a, respectively, and the machine draws the result. z traces the current to the scale k_z and y likewise traces the condenser voltage to the scale $\frac{k_d}{L}$ cm. per volt.

If it is desired to solve the circuit with some arbitrary applied voltage and initial conditions the same scales still apply and the

manner of solution is only slightly altered. Let the new conditions be

$$e = E(t) \text{ applied at } t = 0$$

$$\begin{cases} i = i_0 \\ V_c = V_0 \\ t = 0. \end{cases} \tag{22}$$

The equation of the circuit is then

$$E(t) = L\frac{di}{dt} + ri + \frac{1}{C}\int idt, \tag{23}$$

which being transposed, differentiated once and divided by L becomes

$$\frac{d^2i}{dt^2} = \frac{1}{L}\frac{d}{dt}[E(t) - ri] - \frac{1}{LC}i. \tag{24}$$

If the substitutions (15) are used except that now

$$f_d = k_d\frac{1}{L}[E(t) - ri], \tag{25}$$

the scales are unchanged. f_d is now a function of both t and i, and the method previously outlined for handling a double function is used. Depending on the form of $E(t)$ and the relative magnitudes of $E(t)$ and ri, it may be more convenient to drive the C platen by z and interpolate according to values of t indicated by x, or the reverse. $[E(t) - ri]$ is plotted to the scale of $\frac{k_d}{L}$ centimetres per volt just as ri in the previous case.

To satisfy the initial conditions (22), R is started from the zero position ($x = 0$). z is displaced $k_z i_0$ centimetres, corresponding to i_0 amperes, and βy displaced to $\frac{k_d}{L} V_0$ centimetres representing V_0 volts on the condenser. The machine is then ready to be started.

Variations of the set-up corresponding to other alterations of the problem will be evident when it is observed that the current, voltage, and time all appear directly on the machine plotted to definite scale. It is merely necessary to set the machine to any initial conditions desired and to make the applied voltage a func-

tion of any variable desired, and the corresponding solution will be drawn by the machine.

In the example just given, the coefficients were all assumed constant. This is entirely unnecessary and the introduction of variable coefficients requires very little additional work. For example, let L be a function of current

$$L = F(i) \tag{26}$$

and consider the solution of (24) with the condition (26) added. The equation then becomes

$$\frac{d^2i}{dt^2} = \frac{d}{dt}\left[\frac{E(t) - ri}{F(i)}\right] - \frac{1}{C}\frac{i}{F(i)} \tag{27}$$

and the substitutions (15) with the following changes

$$f_d = k_d\left[\frac{E(t) - ri}{F(i)}\right]$$

$$f_a = k_a\frac{i}{F(i)} \tag{28}$$

$$f_1 = -\frac{k_1}{C},$$

when substituted in (12) give the result

$$\frac{d^2i}{dt^2} = \frac{n\alpha a k_x k_d}{bk_z}\frac{d}{dt}\left[\frac{E(t) - ri}{F(i)}\right] - \frac{n^2\alpha\beta a k_a k_1 k_x^2}{k_z}\left[\frac{i}{CF(i)}\right]. \tag{29}$$

This compared with (27) shows the following to be true for identity

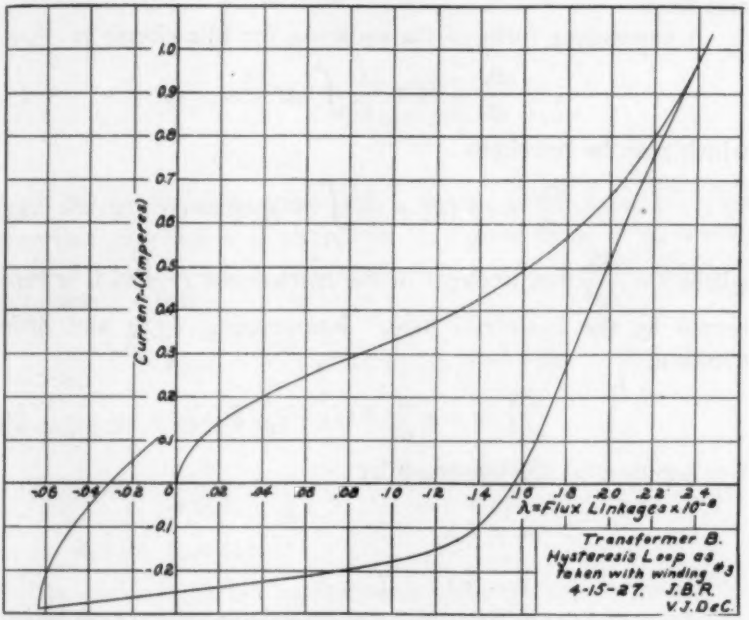
$$\frac{n\alpha a k_x k_d}{bk_z} = 1 \tag{30}$$

$$\frac{n^2\alpha\beta a k_a k_1 k_x^2}{k_z} = 1, \tag{31}$$

which are the same as (20) and (21). Since $F(i)$ is an inductance, k_d still has the dimension of centimetres per unit of volts per henry and the quantity $\left[\frac{E(t) - ri}{F(i)}\right]$ is plotted to this scale. k_a has the dimension of centimetres per unit of $\frac{\text{amperes}}{\text{henry}}$. However, for convenience in setting initial conditions and in reading

final results, the plot which f_d follows can be scaled in terms of volts, and the f_a plot similarly scaled in amperes, since for a given value of volts and amperes, each of these quantities is fixed. Aside from the slight additional labor in plotting the f_d and f_a curves for use in equation (27), it is no more difficult or tedious to solve than the original for (14). Analytically, however, there

FIG. 10.



Static hysteresis loop of the iron core reactor used in example 1. This loop was taken for the range of current covered by the transient.

is no comparison between these two equations, a step-by-step solution being the only method of attack on equations with arbitrarily variable coefficients.

EXAMPLE I.

As a sample of an equation readily solvable on the integraph but exceedingly awkward to handle by any other method, a circuit consisting of an iron core reactor into which a condenser is discharged will be solved in detail, and the solution compared with an oscillogram. While this is electrically very similar to the circuits considered above, in this case the hysteresis loss produces most of the damping of the transient, making it a particu-

larly interesting problem. Also, the R, L, and C circuit in its variations furnishes excellent examples of typical second-order differential equations which are familiar to every one. The values of the circuit parameters in this particular problem are so chosen that the iron is well saturated at the maximum value of current, a case practically unmanageable in analytical form when the hysteresis loop is used to express the relation between current and flux.

A convenient form of the equation for this circuit is

$$\frac{d\lambda}{dt} + ri + \frac{i}{C}\int idt = 0, \tag{32}$$

which may be rewritten

$$\frac{d\lambda}{dt} + rF(\lambda) + \frac{1}{C}\int F(\lambda)\,dt = 0, \tag{33}$$

where $\lambda = \frac{N\phi}{10^8}$ flux linkages in the reactor and $i = F(\lambda)$, is represented by the hysteresis loop. Rearranging (33) and differentiating

$$\frac{d^2\lambda}{dt^2} = -r\frac{d}{dt}F(\lambda) - \frac{1}{C}F(\lambda). \tag{34}$$

For solution on the integraph let

$$\begin{aligned} z &= k_z\lambda & f_1 &= -\frac{k_1}{C} \\ x &= k_x t & f_2 &= 0 \\ f_a &= k_a\lambda & f_d &= -k_d r F(\lambda). \end{aligned} \tag{35}$$

Using the relations corresponding to (16) and (17), substituting in (12) and rearranging

$$\frac{d^2\lambda}{dt^2} = \frac{n\alpha a k_x k_d}{b k_z}\left[-r\frac{dF(\lambda)}{dt}\right] - \frac{n^2\alpha\beta a k_x^2 k_a k_1}{k_z}\frac{F(\lambda)}{C}, \tag{36}$$

which is identical with (34) if the quantities satisfy (30) and (31).

In this problem a condenser of 20.56-mfd. capacity was charged to 100 volts and discharged through a small transformer of which the hysteresis loop of Fig. 10 was taken. Experimentally it is known that the period of this discharge is roughly 0.015 sec. and that the maximum value of current is about 1.0 ampere.

This is a convenience, though by no means a necessity, in the choice of scales. Also the value of λ corresponding to 1.0 ampere is about 0.25.

The following scales are convenient

$$k_x = 2500 \text{ cm./sec.}$$
$$k_z = 75 \text{ cm./unit of } \lambda$$
$$\alpha = 1 .$$

Then

$$nk_d = \frac{bk_z}{ak_x} = \frac{0.00672 \times 75}{1 \times 0.000678 \times 2500}$$
$$= 0.298 .$$

Since the condenser voltage which is set to the same scale on βy as that to which ir is plotted on f_d, a good value of k_d is 0.10 cm./volt approximately, indicating that $n = 2$, or slow speed, is preferable. Then

$$k_d = \frac{0.298}{2} = 0.149 \text{ cm. per volt.}$$

Assuming $\beta = 1$ as a trial

$$k_a k_1 = \frac{k_z}{n^2 \beta a k_x^2} = \frac{75 \times 10^6}{4 \times 1 \times 1 \times 0.000678 \times 6.25}$$
$$k_a k_1 = 0.00442.$$

The current reaches a maximum value of approximately 1.0 ampere, hence a scale for f_a of 15 cm. per ampere is good.

Then

$$k_1 = \frac{0.00442}{15}$$

and

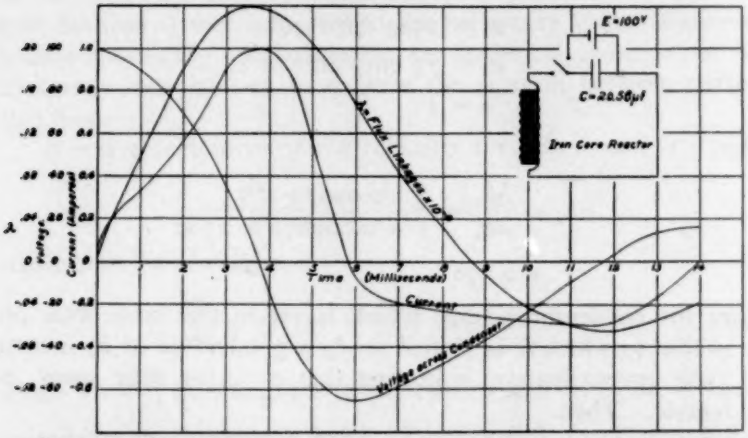
$$f_1 = -\frac{k_1}{C} = -\frac{0.00442 \times 10^6}{15 \times 20.56} = -14.34 \text{ cm.}$$

which is practicable, therefore the scales are satisfactory as chosen.

This completes the selection of scales and after having plotted the hysteresis loop and the ir drop to the scales selected, the

problem is ready for the machine. With the hysteresis loop on platen B and the ir drop curve on platen C, z (representing λ) is set to zero and platen B clamped so that the f_a pointer is at the

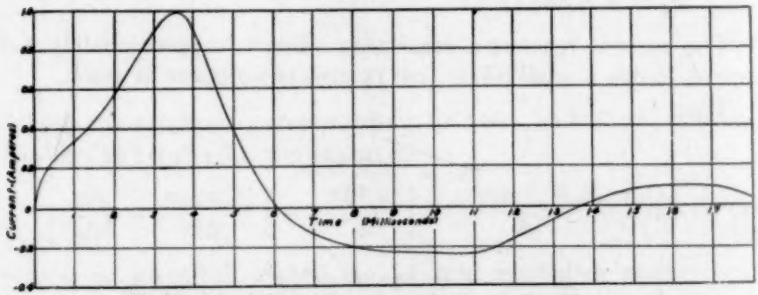
FIG. 11.



Integraph solution of the transient caused by discharging the condenser C into the iron core reactor using the hysteresis loop of Fig. 10 as the magnetization curve of the reactor. The check between the current curve and the oscillogram is rather close considering that the results were calculated using a static hysteresis loop and that the effect of eddy currents in the core was neglected.

zero of λ on the hysteresis loop. The zero of y is found by setting it to the position which makes $\frac{dz}{dx}$ zero when the machine

FIG. 12.



Oscillogram of the transient current in the circuit of Fig. 11.

is running. y is then given a displacement y_0 corresponding to 100 volts potential on the condenser.

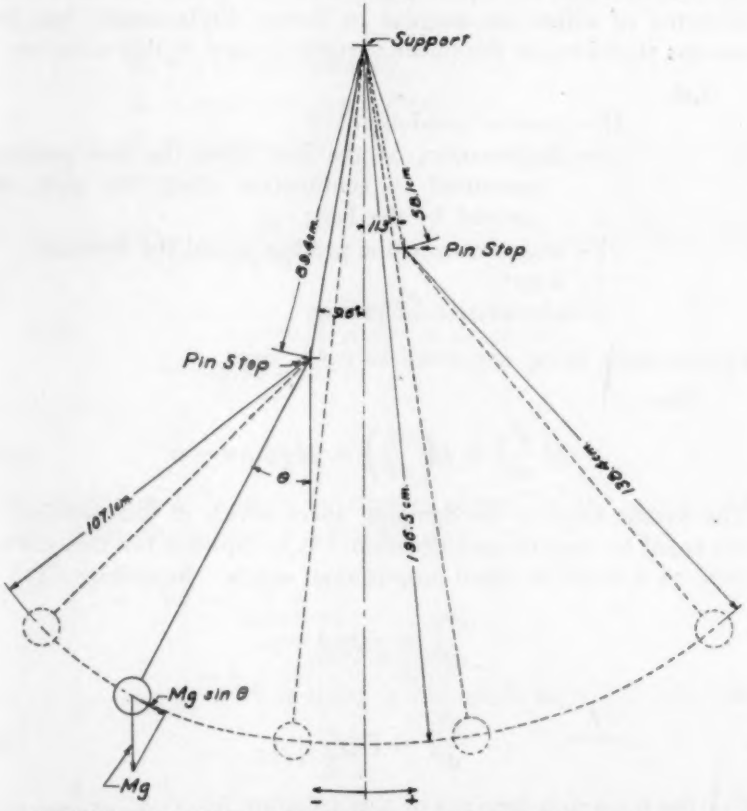
$$y_0 = k_d V_c = 0.149 \times 100$$
$$= 14.9 \text{ cm.}$$

The starting switch is then closed and the integraph draws the results which are shown in Fig. 11. Fig. 12 is an oscillogram of the discharge current taken on the apparatus for which the calculations were made.

EXAMPLE 2.

A particularly simple physical problem somewhat inconvenient in analytical solution, is the pendulum of varying length

FIG. 13.



Multiple length pendulum. Solutions and experimental checks of the period were made for initial displacements of −70.0° and −47.7°. These are sufficiently large amplitudes to make the approximation of $\sin\theta = \theta$ invalid.

shown in Fig. 13. The suspension is of flexible wire so that as the pendulum swings against the stop at either side, the stop serves as a new pivot point. In addition to the multiple length

condition, a large amplitude of swing is used so that the usual simplifying assumption, $\sin\theta = \theta$, is not sufficiently accurate, making a formal solution cumbersome, but causing no complication in the integraph solution. On the machine, a broken line restoring force curve replaces the straight line which would be used for the solution of a simple small amplitude pendulum. The machine solution with an experimental check will be given as another example of the method of using the integraph.

Consider first the equation to be solved. This can be written in terms of either an angular or linear displacement, but for reasons stated below the linear variable is used in this solution.

Let

M = mass of pendulum bob;
s = displacement of the bob from the rest position measured in centimetres along the path described by the bob;
θ = angle between the pendulum and the vertical;
t = time;
g = acceleration of gravity;

all quantities being expressed in c.g.s. units.

Then

$$M\frac{d^2s}{dt^2} + f\left(\frac{ds}{dt}\right) + Mg\sin\theta = 0. \tag{37}$$

The middle term is the damping force which in this problem is too small to require consideration. It is omitted for this reason only, as it could be taken into account easily. Rewriting (37)

$$\frac{d^2s}{dt^2} + g\sin\theta = 0.$$

or

$$\frac{d^2s}{dt^2} + F(s) = 0. \tag{38}$$

For the integraph solution of this equation let

$$\begin{aligned} z &= k_z s \\ f_a &= k_a F(s) \\ x &= k_x t \\ f_1 &= \text{constant}. \end{aligned} \tag{39}$$

Then

$$\frac{d^2s}{dt^2} = n^2\alpha\beta a \frac{k_x^2 k_a f_1}{k_z} F(s), \tag{40}$$

whence

$$\frac{n^2\alpha\beta a k_x^2 k_a f_1}{k_z} = -1. \tag{41}$$

In this problem s reaches a maximum of 200 cm., and the limit of $F(s)$ is 980 cm. per sec.^{2}. The period T is 2½ to 3 seconds.

Let

$$k_z = 0.08 \text{ cm./unit } s$$
$$k_a = 0.015 \text{ cm./unit } F(s)$$
$$k_x = 15 \text{ cm./sec.}$$
$$n = 2$$

$$\alpha\beta f_1 = -\frac{k_z}{a k_x^2 k_a n^2}$$

$$= -\frac{0.08}{0.000678 \times 225 \times 0.015 \times 4}$$

$$= -8.74 \text{ cm.}$$

If $\alpha = \beta = 1$

$$f_1 = -8.74 \text{ cm.}$$

The velocity as a function of time is plotted by the y index to a scale determined from (11), (8), and (39).

$$\beta y = \frac{k_z b}{k_x n \alpha a} \frac{ds}{dt}$$

$$= \frac{0.08 \times 0.00672}{15 \times 2 \times 1 \times 0.000678} \frac{ds}{dt}$$

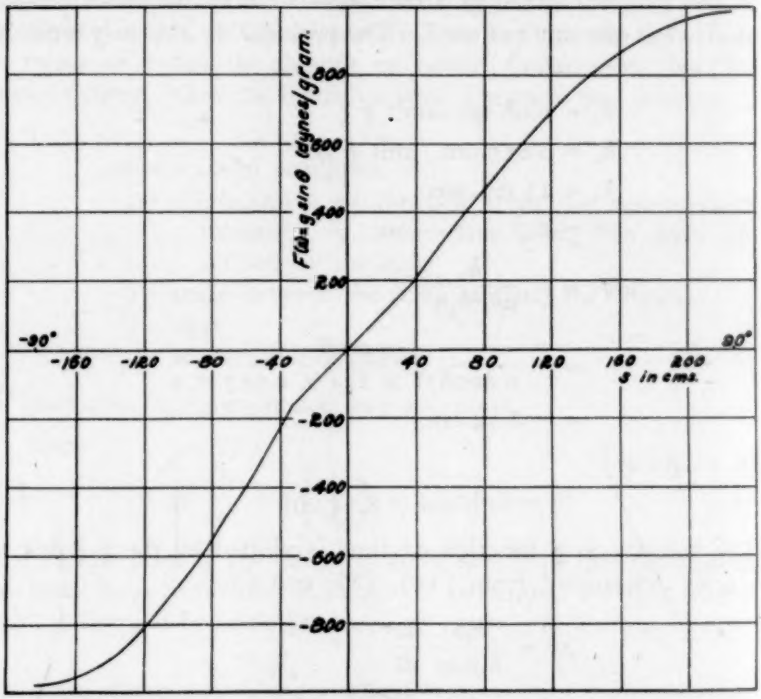
$$= 0.0264 \frac{ds}{dt}.$$

Two amplitudes of swing were calculated and checked experimentally, the first with $-70.9°$ initial displacement, the second with $-47.7°$. With the machine set up as before, z is given a displacement in the first case of $-0.08 \times 147.8 = -11.81$ cm. and in the second of $-0.08 \times 104.0 = 8.32$ cm., -147.8 cm. and

-104.0 cm. being the arcs corresponding to the above angles of $-70.9°$ and $-47.7°$, respectively. Since $\frac{ds}{dt} = 0$ at starting, y is started on zero.

The results expressed graphically and the experimental check are shown in Fig. 14. While the accuracy is not that which is

FIG. 14.



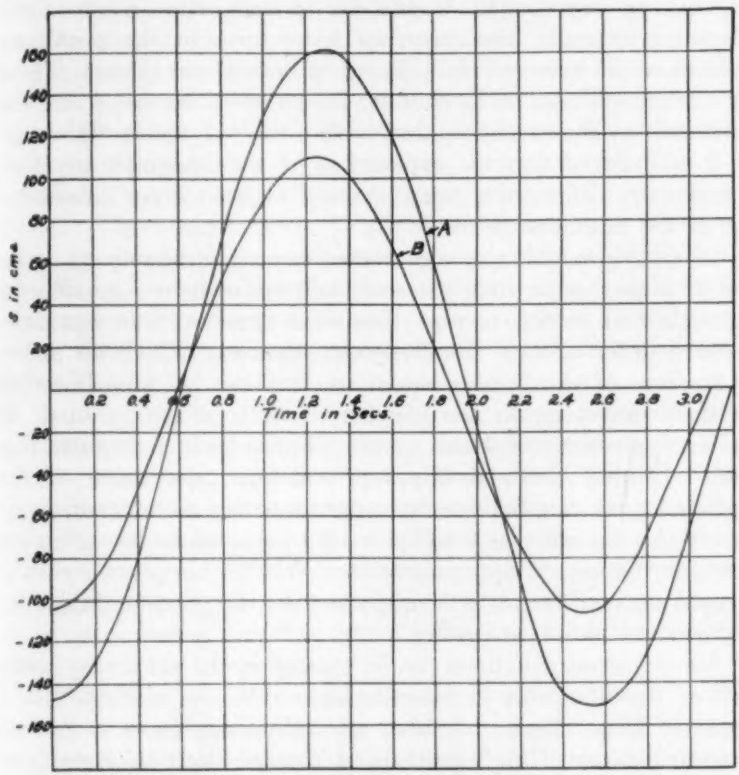
Restoring force curve for the pendulum shown in Fig. 13. The force $F(s)$ is plotted as a function of s, s being the displacement of the pendulum bob from the rest position measured in centimetres. The breaks in the curve occur where the pendulum length changes.

ordinarily associated with a pendulum, it must be remembered that the pendulum is an extraordinarily precise piece of apparatus, and that the accuracy of the solution secured on the integraph is adequate for many engineering problems, especially those involving empirical data.

As mentioned above, the problem can be worked with θ as the independent variable, but it must be remembered that the angular *velocity* is discontinuous at the point where the length of pendu-

lum changes and that a term must be included in the differential equation which states this physical fact. This point must be carefully watched when dealing with problems involving discontinuities. It is perhaps easier when practicable, in a problem

FIG. 15.



Solution of pendulum problem expressed as displacement *vs.* time.

Curves.	*Initial Displacement.*		*Period.*	
	Degrees.	*s* in cm.	Calc.	Observed.
A	−70.9°	−148	2.58 sec.	2.565 sec.
B	−47.7°	−104	2.50	2.500

It will be noticed that the swing is unsymmetrical due to the non-symmetry in the set-up.

involving discontinuities, to pick as the independent variable a quantity which has discontinuities only in the highest order derivative appearing in the equation.

The examples given above serve merely to indicate some of the ways in which the integraph can be applied to the solution of

differential equations. For the sake of clearness the functions involved in the problems chosen were relatively simple, whereas in several problems which have actually been solved by the machine, much more complicated functions have been used. It is to be noted that some functions which are inherently simple may not be represented readily by formal expressions. Consider, for example, the restoring force term in the pendulum problem or the hysteresis loop in the saturated iron circuit. These are not complicated as functions, nevertheless we have not the mathematical equipment to deal with either of them effectively.

It is believed that the application of the integraph to other second-order differential equations will be seen from consideration of the examples given.

While the machine as constructed can solve directly no equation of higher order than the second, it is frequently possible by the method of successive approximations to solve either equations of third order or two simultaneous equations of second order. In the case of third-order equations, this can be done if either the third derivative or the constant term is relatively small so that an approximate solution can be obtained which neglects this term. Having obtained this approximation, the value of the neglected term can be obtained approximately as a function of one of the variables and added in the equation as an arbitrary term for the second approximation. Where this process results in rapid convergence, as it frequently does, the given higher order equation can be solved readily.

Simultaneous equations can be treated in the same way when each of the equations is of such form that one variable has a relatively large effect, the other variable entering as a sort of correcting factor. The equations of coupled electrical circuits or mechanical systems where the coupling is not large furnish excellent illustrations of this type of simultaneous equations solvable by successive approximations. While this may seem laborious and approximate, it is to be remembered that one solution can be run in eight minutes or less, and that making several trials is a relatively short process.

The foregoing presents some of the possibilities of the integraph in the solution of problems for which it was definitely constructed, and an outline of methods by which it often can be used to handle indirectly problems which are nominally beyond

its scope. Further applications and extensions of its possibilities frequently appear when other forms of equations are presented for solution.

ACCURACY OF THE INTEGRAPH.

Two principal sources of uncertainty of results exist. The first includes the error in plotting the functions used, and in reading the curves drawn by the machine. The second consists of the errors, random or systematic, which occur in the machine itself. In general, errors of the first group are less than those in the integraph, since a carefully made plot on the scales which it is practicable to use is accurate to somewhat less than 1 per cent., which is the order of magnitude of the errors in the machine itself. The human factor involved in following a curve with an index is relatively small since, in every case, a function thus introduced into the machine is integrated, a process which is little affected by local deviations from a given function, provided these deviations are approximately equal on both sides of the curve.

Among the inherent machine errors, those discussed above are the most serious, viz., lag errors due to backlash, to lag in the servo-mechanisms, and to the watthour meter inertia. These, from their nature, tend to become cumulative in the solution of many second-order equations, and hence are serious if they are allowed to occur. Their elimination merely requires recognition of their existence and adequate provision for either removing the cause or canceling the effect by compensation. Both these methods are used on the integraph.

Non-linearity of the potentiometer slide wires is another source of trouble which can be rendered inappreciable by careful construction. It is essential, of course, that the currents through the watthour meter coils be strictly proportional to the displacement of the indexes on the slide wires. Provided the current drawn by the watthour meter coil is small in comparison with the main slide-wire current, this linearity can be secured by careful winding of the slide wires with wire of uniform diameter and resistivity.

Heating of the watthour meter coils increases their resistance, decreasing the current through them for given voltages applied. Warming the coils by passing current through them for several minutes before using the machine decreases this error to a small

quantity, the heat capacity of the coils being sufficient to smooth out the variations in heating during a run.

In order to keep the calibration constant, it is necessary that the frequency of the alternating-current power driving the synchronous motors be held at a definite value throughout a run. The direct-current voltage supplied to the slide wires must also be held to the same accuracy that is desired in the final results.

Each of the sources of error mentioned must be considered to secure the most accurate results. For any particular run it is difficult to predict the exact magnitude of the error, but experience shows that the results are consistently within 2 or 3 per cent., and except when unfavorable gear ratios are used accuracy within 1 per cent. is to be expected. This is not precision work, but seldom are the data upon which solutions are based certain to even this accuracy. It follows that this integraph has sufficient accuracy for the majority of problems encountered in engineering work.

Where greater accuracy is required, it is frequently practicable to solve not the original equation, but a difference equation, giving as a result the difference between some approximate analytical solution and the true solution. For example, if a given analytical solution approximates the true solution within 10 per cent., a difference equation giving the correction to 1 per cent. will yield a final result accurate to 0.1 per cent. This method is very powerful in certain cases.

SUMMARY AND CONCLUSIONS.

Formal mathematics is the foundation of the physical sciences and engineering both in the formulation and solution of physical relationships. Because of the imperfection of our formal mathematical tools, however, many analytical statements of problems are of very restricted value because they cannot be solved readily, if at all. Particularly is this true where equations are encountered involving coefficients which are arbitrarily variable, or have discontinuities or are multivalued. In this paper has been described an integraph which solves differential equations of precisely this type when the equations are of first or second order.

This integraph employs two integrating devices which perform successive integrations on the variables introduced, these

operations being an essential part of the solution of a second-order differential equation. The first stage of integration uses a modified Thompson direct-current integrating watthour meter which gives the integral of the product of two independent functions, one of which may itself be the sum of two independent functions. The result of this first integration is again integrated by a Kelvin disc and wheel integrator. By a suitable mechanism each of these results is plotted automatically, and through coupling devices the original functions may have as variables either the independent variable, the result of the first integration, or the result of the second integration. It is possible and practicable thus to solve a total second-order differential equation of a form so closely approaching the most general form that exceptions are very rare.

By various devices the errors inherent in such a mechanism have been removed or reduced to safe values. No power is drawn from the integrating devices themselves, all the work being done by servo-motors controlled by relays actuated through contacts on the integrators. Also friction torque in the integrators has been rendered ineffective by making the net motion between the integrating element and its main bearing zero. Errors due to lag in the mechanism have been removed at their cause or canceled by compensation. Mechanically the machine is sufficiently refined to give engineering accuracy in solutions performed with it.

Several examples of solutions which have been performed are given in detail. These are of sufficiently striking nature from the analytical mathematical point of view to demonstrate the power of the integraph where present formal methods are cumbersome.

Engineering can proceed no faster than the mathematical analysis on which it is based. Formal mathematics is frequently inadequate for numerous problems pressing for solution, and in the absence of radically new mathematics, mechanical solution offers the most promising and powerful attack wherever a solution in graphical form is adequate for the purpose. This is usually the case in engineering problems.

A Careful Measurement of the Ratio of the Electrical Charge of an Electron to Its Mass at Rest by the Method of H. Busch. F. WOLF. (*Ann. der Physik*, No. 14, 1927.)—The plan usually employed for the determination of this quantity has been to measure the deflection produced by a magnetic field upon electrons and, concomitantly, to measure either the deflection due to an electrostatic field or the discharge potential itself. From these results the ratio can be computed. Bestelmeyer, after a searching examination of the values obtained by sundry investigators, gives 1.76×10^7 electromagnetic units/gram as the most probable value, with an accuracy of 1 per cent. More recently Gerlach calculates the most probable value to be 1.766. He used selected results and these varied from the mean by only 2 or 3 parts per thousand. A criticism made by Bestelmeyer, however, still holds, that, in many cases, the value given finally by an author is obtained from separate values that depart considerably from the end-result. For this reason it seemed good to make a fresh determination of this important natural constant, recourse being had to recent experience and to improved ways of measuring.

In Busch's method a slightly divergent bundle of cathode-rays is sent through a uniform magnetic field with its axis parallel to the lines of force. Electrons moving along a line are not deflected but those making an angle with the lines are caused to move in advancing spirals having lines of force for axes. By the time an electron has made just one turn about the line it is again upon the line of force which passed through it when it began its course. The time of one such rotation is independent of the radius of the spiral and of the velocity of the electron, so that all electrons from the same point after the same interval are again found on the same line of force. The distance from the starting-point to the point where the electrons reassemble does depend upon their speed parallel to the lines of force, but if the path of no electron makes more than a small angle with the field and from the fact that the potential applied gives the same speed to all electrons, it follows that this distance is the same for all. A fluorescent screen was used to catch the electrons and to determine when they formed a clear image of the cross-wires in the aperture through which the particles enter the magnetic field. In practice the focussing was done not by moving the screen back and forth, but by varying the strength of the magnetic field. The distance from diaphragm to screen was about 30 cm. The speed of the electrons was about .13 of that of light.

Great care was exercised to eliminate or to correct for all sources of error. The final value is 1.767 electromagnetic units/gram. This is the mean of 70 determinations. Two of these differ from the mean by .2 per cent., three by .13 per cent., while the remaining 65 depart from it by less than .1 per cent., such is the accuracy of the method. G. F. S.

THE ELECTROSTATICS OF THE THUNDERSTORM.

BY

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I. GENERATION OF CHARGES AND POTENTIALS.

Instructive Analogues.—An instructive analogue of the electrical phenomena of the thunderstorm, and one which exhibits very beautifully the electrostatic features of the processes going on during a thunderstorm, is given by the well-known experiment of the generation of charges and potentials by rubbing together two dissimilar substances, *i.e.*, the fundamental experiment of frictional electricity.

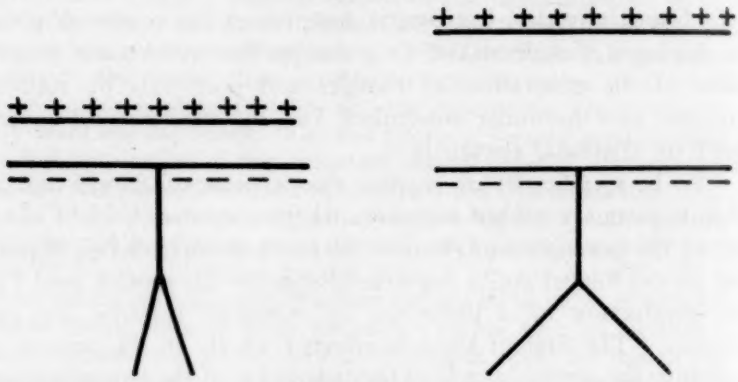
To be specific, let us suppose that a piece of flannel and an ebonite plate are rubbed together and then separated. The result is (1) *the generation of electric charges,* a positive charge appearing on the flannel and a negative charge on the ebonite, and (2) *the production of a difference of potential between the two charges.* The first of these is effected wholly in the process of rubbing; the second largely in the separation of the two substances with their adhering charges.

In order to explain the phenomenon we have to assume merely the existence of a weak source of e.m.f. between the two rubbing substances, this source of e.m.f. then produces the initial *electrical* separation, *i.e.*, generates the charges; the large potential difference between the two charges is then accounted for by the *mechanical* separation of the two charges when the ebonite and flannel are drawn apart; in order to effect this mechanical or gross separation, work must be done, *i.e.*, energy must be expended, against the attractive forces of the two charges, and this work appears as increased electrical energy and, consequently, increased potential difference between the two charges of the system.

The process can be illustrated in another way by the condensing electroscope experiment of Volta. In this case we use two conducting plates separated by a very thin layer of dielectric and charge them with a battery. The two plates take the rôle of carriers only, as did the ebonite and flannel, after the rubbing

process had been completed. In place of rubbing and so making use of the frictional electromotive force, we now apply a battery directly, let us say of the order of a volt. We produce an electrical separation as before. If an electroscope is now connected to the lower plate, the leaves show no divergence, since one volt is not sufficient to affect them. If, however, the upper plate is raised, work is done against the attractive forces of the two charges, this work appears as increased electrical energy, and

FIG. 1.



therefore increased potential of the system, and the electroscope leaves diverge. (Fig. 1.)

Quantitatively, we have as the energy W_1 and W_2 of the system before and after lifting off the upper plate, respectively:

$$W_1 = Q_1 V_1/2\,,\ W_2 = Q_1 V_2/2\,,$$

the amount of energy $(W_2 - W_1)$ being supplied in the act of lifting off the plate.

Since the energy of a condenser is also given by $CV^2/2$, we have also $W_1 = C_1 V_1^2/2$ and $W_2 = C_2 V_2^2/2$ and, combining these equations, we have finally

$$W_2/W_1 = V_2/V_1 = C_1/C_2\,.$$

Looking at the experiment now from the point of view of *capacity*, it is easily seen that lifting the plate vertically is not essential; it can just as well be moved horizontally (or any other way) as long as the capacity is changed.

A very beautiful demonstration of these ideas, particularly

of the equation above, is to take a variable air condenser,[1] connect across it an electrostatic voltmeter, and charge the condenser when its capacity is a maximum to a definite voltage, then turn the dial toward minimum capacity: The voltage is observed to rise in inverse proportion to the capacity.

If the dial is then turned back to the original setting, the voltmeter needle goes back toward zero, energy being now given up and the voltage falling in proportion. The process can be repeated as often as desired.

Application to Electrical Storms.—Turning now to the thunderstorm problem it is seen that if we consider two clouds or two portions of the same cloud as the two plates of a condenser, in a special case a cloud as one plate, the earth as the other, or abstract entirely from the condenser plates and simply consider the two charges, since the clouds act merely as carriers of the charges, we require only (1) a source of initial electrification and (2) a mechanism of gross separation, to explain the genesis of the charges and the potentials during a thunderstorm.

With regard to the source of the primary or initial separation, we have the experimental result that water falling with sufficient velocity in an uprushing current of air becomes electrified,[2] the air negatively, the water-drops positively, and also that an essential feature of the thundercloud is a strong uprushing current of air. Taking this for granted, let us see what the consequences are. We have (1) an initial source of electrification and (2) a mechanism of gross separation, *i.e.*, the condensing electroscope experiment on a large scale.

The energy required to effect the gross separation of the charges, on this view, comes from the kinetic energy of the wind and the gravitational energy of the falling rain.

Of course the processes of the generation of charges and the separation of charges go on simultaneously and continuously in the cloud, that is to say, the charges are not first completely formed, and then separated, as was the case in the experiments cited above; the action of the storm cloud is more like the action

[1] The condenser must be well insulated and the plates sufficiently far apart so that a spark does not go over as the voltage rises; a standard 1000-mmf. condenser placed over P_2O_5 does very well. The potential of two 45-volt B batteries is easily transformed to 1500 volts.

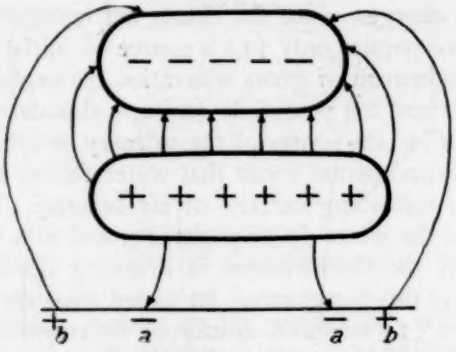
[2] G. C. Simpson, *Memoirs*, Indian Meteorl. Dept., Simla, **20**, pt. 8, 1910.

of a frictional static machine, which continuously generates and separates the charges. If, however, two clouds already charged with equal and opposite quantities of electricity move apart, then we do have the condensing electroscope experiment on a large scale.

II. INDUCTION OF CHARGES.

Induction of Charges by Separation of Charged Clouds.—As a result of the process going on in a thundercloud, we will then have an accumulation of one charge (the negative) in the upper portion of the cloud, an accumulation of an equal and opposite charge in the lower portion, provided of course that the process described can go on without any of the charged rain

FIG. 2.

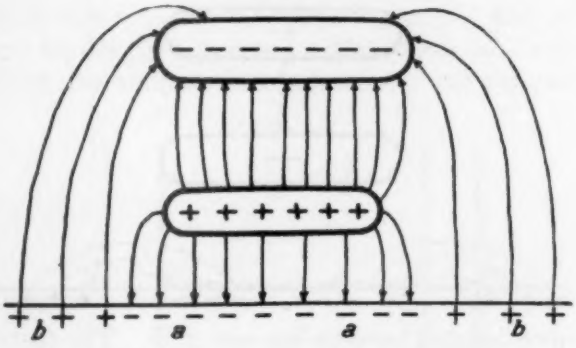


actually falling to earth, and a large difference of potential between the upper and lower portions of the cloud.

If the charges in the cloud are relatively close together compared to their distance from ground, then most of the lines of force pass directly from the lower portion of the cloud to the upper, only a few stray lines reach the earth (Fig. 2). However, when the charges become widely separated either vertically (Fig. 3), or horizontally (Fig. 4), then a portion of the flux passing from cloud to cloud reaches to earth, and, since the earth is conducting, induced charges appear at the earth's surface (at the conducting, *i.e.*, moisture level). But even in the case represented by Fig. 3 there is little tendency to strike to earth; it is only for the case represented by Fig. 4 that a strike to earth might occur.

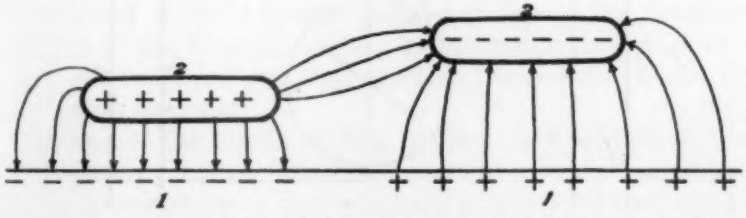
Secondary Sparks Produced by Induced Charges.—Theoretically, however, the induced charges in these two cases could produce secondary sparks if a lightning stroke passed from cloud to cloud. When this occurs, the primary charges in the cloud are suddenly neutralized and the induced charges on the earth previously "bound" are now left "free." Stress will then imme-

FIG. 3.



diately appear between these two charges as represented in Fig. 5, and they will rush to recombine. If the surface earth of the earth, together with all objects on it, were a good conductor, this recombination could be effected almost instantaneously without any appreciable secondary effect; however, since this is not always the case, pools of this charge may be left isolated and for the moment at high potential relative to earth, so that secondary

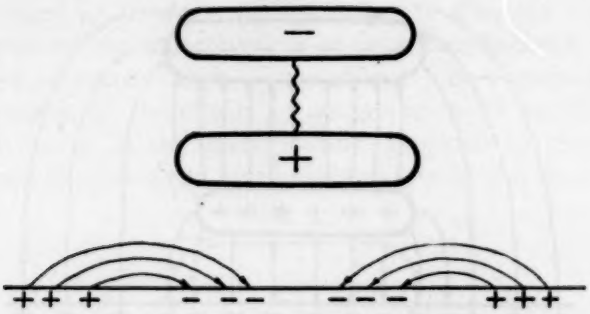
FIG. 4.



sparks may occur. A well-known example is that of the transmission line, which is connected to ground through poor conductors (the insulators) which permit a portion of the induced charge to creep out on the line, a lightning flash from cloud to cloud "releases" this charge, which, if sufficiently strong, breaks over the insulators to ground.

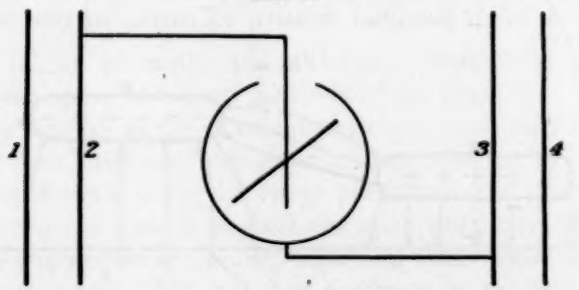
Now it is to be noted that the secondary spark in this case is not to be interpreted in the sense that the line is dividing its charge with the earth; the voltage between the line and ground is produced by virtue of the fact that a charge equal in value but opposite in sign to the charge on the line concentrates in the ground area under the line after the lightning flash; as a result,

FIG. 5.



electric stress appears between line and earth. The charge which, immediately after the flash, concentrates on the ground area under the line was previously in the ground area which carried the induced charge of opposite polarity to that in the zone around the line. For example, if the line is located in the region *b* of Fig. 3 and carries a charge $+q$, then, after a flash from cloud to cloud,

FIG. 6.



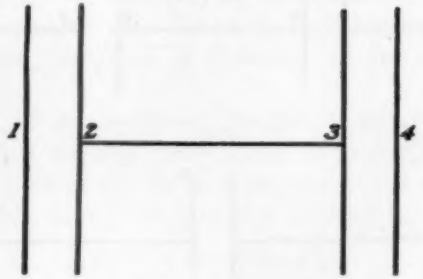
a charge $-q$ rushes from the region *a* over to the ground area under the line, consequently we will have the line raised to a potential V above ground given by $V = q/c$, where c is the capacity of the line.

Whether, however, the *induced* charges under these conditions, *i.e.*, cloud-to-cloud flashes, are sufficiently strong to

produce dangerous secondary sparks is very questionable; it is likely that only precipitated *primary* charges, as we shall see later, are sufficiently intense to produce dangerous secondary sparks.

This phenomenon of the release of induced charge can be illustrated again by a simple condenser analogue. For this purpose two condensers are connected in series with an electrostatic voltmeter as shown in Fig. 6. The voltmeter being short-circuited, an e.m.f. is applied between plates 1 and 4, *i.e.*, the condensers are charged in series. The source of e.m.f. is then disconnected, the voltmeter short removed, and the two outside

FIG. 7.



plates 1, 4 connected together. The voltmeter now shows a large deflection, in fact a voltage approximately the same as that originally applied across the two condensers now appears across the plates 2, 3.

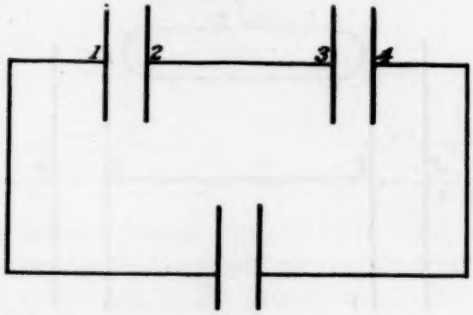
Discharge of Induced Charges.—In the case represented by Fig. 4 it would be possible for a strike to earth to occur from either cloud to the opposite induced charge. The condenser analogue of this situation is given by two condensers (Fig. 7) in series, the two outer plates representing the two clouds, the two inner plates the earth.

If one of the clouds of Fig. 4 discharges to earth it does not necessarily follow that the other must also discharge, since we can discharge one of the condensers of Fig. 7 without appreciably changing conditions in the other. Actually, of course, unless the clouds are very widely separated relative to their distance from ground, we still have some lines of force through the air from one cloud to the other, the condenser analogue being really that of Fig. 8. In this case a discharge from cloud 3 to earth might throw a sufficient portion of the charge originally in cloud 3,

namely, that portion which gives rise to the lines through the air from cloud to cloud, directly under cloud 2, and so raise the stress, if it were already near the breaking point, to the breaking point. But very likely this rarely occurs in practice.

It might also happen that instead of discharging to earth through the medium of a lightning flash that one of the charges of Fig. 4 is carried to earth by the cloud falling in the form of rain; in this case it is readily seen that the final result is the same as for the case just discussed, for example, if cloud 3 falls as

FIG. 8.



rain, the condenser system formed by the cloud 3 and the earth area under it is discharged.

It is interesting to note that if either of the clouds, cloud 3 for example, precipitates its *charge in the form of rain* or even discharges it instantaneously by a lightning flash, the *whole* of this charge does not *suddenly* appear under cloud 2 and *suddenly* raise the stress in that region; stress and charge *due to induction* are already present under cloud 2 before the discharge takes place from cloud 3, the flash or precipitation only transfers the *excess* (that corresponding to the cloud-to-cloud flux) to the earth area under cloud 2. In the case of a flash, the excess is suddenly transferred; in the case of precipitation, it is slowly transferred. There will, however, always be an excess, since an induced charge can at most be equal to the inducing charge. On the other hand, when the cloud-to-cloud flux is very large, *i.e.*, when the excess of charge is large, then the lightning discharge, if it occurs at all, will be apt to place from cloud to cloud, rather than from cloud to earth, so that in none of these cases is there a sudden *large* application of stress to the earth area.

Quantitative Theory of Induced Charges.—Let us now consider somewhat in detail the case (Fig. 4) where all *primary* charges remain in the clouds, *i.e.*, cloud 3 has a charge $-Q$, cloud 2 has a charge $+Q$, and the earth 1 has zero charge.

If the clouds and the earth's surface were good conductors, we could put for any position of the clouds:

$$Q_1 = a_{21}V_2 + a_{31}V_3 = 0 \tag{2}$$

$$Q_2 = a_{22}V_2 + a_{32}V_3 = +Q \tag{3}$$

$$Q_3 = a_{23}V_2 + a_{33}V_3 = -Q \tag{4}$$

where the Q's and V's represent, respectively, the charges and potentials of the bodies indicated by the subscripts, any a_{ii} represents the coefficient of capacity of the body indexed i, and any a_{ij} represents the coefficient of induction of the bodies indexed i and j.

If now we let φ_{21} represent the flux passing from cloud 2 to earth, φ_{31} that passing from cloud 3 to earth, and further let q_2 and q_3 represent the induced charges at the earth's surface due to the clouds 2 and 3, respectively, then we have also:

$$q_2 = -\varphi_{21} = a_{21}V_2 \tag{5}$$

$$q_3 = -\varphi_{31} = a_{31}V_3 \tag{6}$$

combining (2), (3), (4), we have:

$$V_2 = +Q(a_{33} + a_{32})/(a_{22}a_{33} - a_{23}a_{32}) \tag{7}$$

$$V_3 = -Q(a_{22} + a_{23})/(a_{22}a_{33} - a_{23}a_{32}) \tag{8}$$

whence

$$q_2 = \frac{a_{21}(a_{33} + a_{32})}{(a_{22}a_{33} - a_{23}a_{32})}Q \tag{9}$$

$$q_3 = \frac{a_{31}(a_{22} + a_{23})}{(a_{22}a_{33} - a_{23}a_{32})}Q. \tag{10}$$

If the clouds separated until their distance apart was large compared to their distance from ground, we would have $a_{23} \to 0$, $a_{21} \to -a_{22}$, $a_{31} \to -a_{33}$, from which we see that simultaneously $q_2 \to -Q$ and $q_3 \to +Q$.

In other words, only if the clouds become widely separated do the magnitudes of the induced charges at the earth's surface approach that of the primary charges in the clouds.

As the clouds move apart there will be then a continuous

flow of charge, *i.e.*, an electric current, from the earth area under the cloud 3 to the earth area under cloud 2, and if we denote this current by i_{23} we have directly:

$$i_{23} = -\frac{\partial q_2}{\partial t} = +\frac{\partial q_3}{\partial t} = \frac{-\left[\frac{\partial a_{21}}{\partial t}\frac{\partial a_{31}}{\partial t}\right]Q}{\left[\frac{\partial a_{22}}{\partial t}\frac{\partial a_{33}}{\partial t} - \frac{\partial a_{23}}{\partial t}\frac{\partial a_{32}}{\partial t}\right]}. \quad (11)$$

In connection with the above equations it is to be noted that while equations (9) and (10) apparently give a different value for the ratio V_2/V_3 than that obtained from (2), the two values are actually identical; for under the conditions chosen, namely, only three bodies in the field and the total charge of these bodies equal to zero, we have the relations:

$$-a_{22} = a_{12} + a_{32}$$
$$-a_{33} = a_{13} + a_{23}.$$

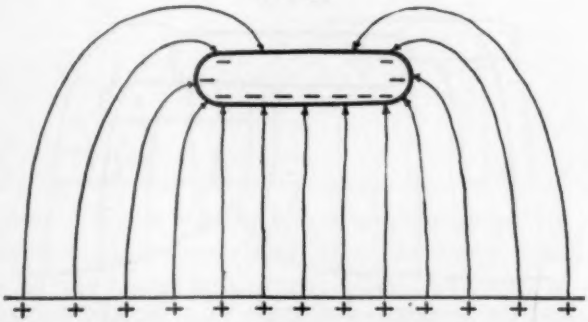
It is readily seen from the above equations that the induction current can vary only as the quantities a_{ij} vary, and the time variation of these quantities in turn is determined by how fast the clouds change their relative positions. This shows that even by the process of induction large charges cannot be impressed instantaneously on the earth's surface.

III. PRECIPITATION OF CHARGES.

This brings us to a consideration of those cases where one of the primary charges is actually carried to earth *by the rain*, and parenthetically it may be stated that very likely these are in the majority in nature, for while theoretically, as we have just seen, it might be possible for a stroke to earth to occur without any of the primary charges formed in the cloud being precipitated to earth, it is also very likely that such cases (charges impressed entirely by induction) are of rare occurrence in practice. Very probably the majority of lightning strokes *to earth* are caused by the direct precipitation of one of the *primary* charges. Ordinarily on Simpson's view this will be the positive charge, since it is associated with the large drops in the base of the cloud, although the negative might be carried away as in Fig. 4, and subsequently fall to earth. If the positive charge has previously been carried to earth, then the precipitation of the negative results

in a reduction of the electric stress, if not, then it results in an increase of electric stress. The ordinary case, and very likely the most frequently encountered case, will be the production of the two charges in an overhanging cloud, the negative in the upper portion, the positive in the lower, and the precipitation of the positive, the negative remaining in the cloud or upper atmosphere. In this case during the process of formation (Fig. 2) we would have originally a weak negative gradient (*i.e.*, lines of force ending on the earth) directly under the cloud (at *a*) due to the stray field, a weak positive gradient some distance out (at

FIG. 9.



b); both of these, as the positively charged rain fell, would gradually be converted to strong positive fields (Fig. 9) until the resulting stress became great enough for a breakdown, *i.e.*, a lightning flash, to occur, when the field would presumably be reduced practically to zero and the process begin over again.

It might also happen that the precipitated charge is not sufficient to effect a breakdown, and that the negative primary charge is carried away by the wind, in which case the precipitated positive primary charge would follow along the ground, presumably at the conducting, *i.e.*, moisture, level. In this case lightning would not occur until favorable conditions for a breakdown, for example, a concentration of flux due to the proximity of a hill as in Fig. 10, presented themselves.

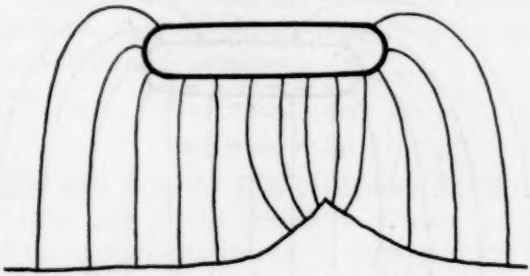
An important point is to be noted in this connection, namely, if the negatively charged cloud should approach the ground, either by dropping as a whole or approaching a hill, there would be reduction in the *potential difference* between cloud and earth, however, there might be local *increases of stress,* so that a break-

down might occur. However, the available voltage would be much less than if the cloud discharged from a higher altitude. To see this we have merely to recall the condensing electroscope experiment, in particular that the *approach* of opposite charges results in a *reduction* of potential, just as the *separation* results in an *increase* of potential.

In the limit if the charged cloud as a whole falls to earth in the form of rain, the opposite charges are reunited and the difference of potential reduced to zero.

And this brings us to an important point, often overlooked,

FIG. 10.



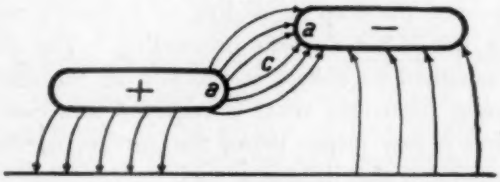
namely, that the systems of charges of a thunderstorm must be treated as a system of charges under the influence of their own field, rather than a system of bodies connected to sources of constant e.m.f.

The power of the machine which generates the electrical energy of a thundercloud is relatively small, but the energy available in the cloud, *i.e.*, the power developed by a lightning flash, may be relatively large, due to the storing up of charge, much as in a static machine the available power is small but relatively large amounts of energy are stored up in the Leyden jars. In fact, in a lightning stroke all the immediately available energy is that stored up in the electric field, there is no powerful agency behind the stroke tending to maintain the voltage.

It is interesting to note in connection with Fig. 10 that the concentration of the flux when a cloud approaches a hill is not so readily effected for an actual cloud as it would be if the cloud were a good conductor. A concentration of flux, of course, means a concentration of charge in certain areas (in the example given these are the area of the hilltop and the area of the under-

face of the cloud immediately over it); however, since the cloud is not a good conductor, the flow of charge into this region, *i.e.*, this concentration of charge, cannot readily take place. An analogy is provided by the case of a charged conducting sphere lowered over a conducting plane. As the sphere is lowered, the charge on it concentrates on the hemisphere toward the plane, in fact it concentrates on an area round the lower pole, likewise the charge on the plane will be drawn in toward a point directly under this pole and, as the sphere is lowered, eventually due to the concentration of flux, *i.e.*, increase of stress, a spark will pass

FIG. 11.



to the plane. If the sphere were a non-conductor (in the ideal case if it were *rigidly* electrified) then the charge could not concentrate on the lower face of the sphere, although it would to some extent on the plane. Finally, if the plane were also rigidly electrified, there would be no concentration of charge and very little concentration of flux, hence a spark would not pass until the sphere was brought very close to the plane.

The same principle applies to the separation of two clouds. If they separate tangentially, then as they come into the position of Fig. 11, theoretically, if they were good conductors, all the charge would run toward the adjacent ends *aa*, raising the field in the space *c* to very large values, so that a breakdown would be apt to occur. However, since clouds are actually poorly conducting bodies, it is very likely that the clouds can separate more rapidly than the concentration of stress can increase, due to this cause. It may also be pointed out that the simultaneous increase in *potential* due to the separation has little to do with the increase in *stress*, since it does not change the total number of lines passing from one cloud to the other.

Of course it is to be noted with regard to Figs. 2 and 3 that the actual distribution occurring in nature will not be purely

vertical or horizontal, but will be determined by the actual air currents in a thundercloud; similarly the distribution on the ground will be determined partially by how and where the rain happens to fall and partially by subsequent creepage of charge in response to unevenness of ground, namely, the effect of the geometrical configuration of the surface. Induced charges (such as those pictured in Fig. 4) will presumably make use of the permanent underground water level as the conducting path over which the separation is effected, however, subsequently they also will tend to creep toward the surface of the earth directly under the clouds. In any case then, the charge will be on the *surface* of the earth—and the deep grounding of lightning rods therefore appears as a somewhat dubious proceeding. The charge which the rod is assumed to collect is literally at the surface of the earth; it would seem that what is required is a conducting network of wires a few inches below the surface of the earth and radiating out from the lightning rod.

Relation between Polarity of Rain and Polarity of Cloud.—Going back now to the consideration of the precipitation of charges by rain, it is interesting to ask whether theoretically there is any connection between the electric (convection) current carried down by the rain over a given area and the resulting potential gradients at points in that area.

A common error in this connection is to suppose that the sign of the charge on the rain is an indication of the polarity of an overhanging cloud. As a matter of fact, this is not the case. In the usual process of the generation of charge on a cloud, the charging of the cloud *negatively* with regard to earth is accomplished by the precipitation of *positively* charged rain. On Simpson's view of the process of generation of charge it could also happen, although less frequently, that a cloud would be charged *positively* with respect to earth by the precipitation of *negatively* charged rain. These two instances are merely illustrative of the fact that it is possible to charge a body with electricity of one sign by removing from it that of the opposite sign. From Simpson's view of the process going on in the cloud, or any view which puts the seat of e.m.f. in the cloud, it is readily seen that the cloud *as a whole* is initially electrically neutral with

regard to the earth and remains so until one of the charges is precipitated or far removed.

On the other hand, if a cloud is strongly negative with respect to earth, *i.e.*, if the positive charge has previously been precipitated or far removed as in Fig. 4, then the polarity of the falling rain is an index to the polarity of the charge on the cloud.

Relation between Polarity of Rain and Sign of Potential Gradient.—Turning now to the potential gradient relation we may note that in the first case considered above, namely, that case where by the precipitation of electricity of one sign, an *excess* of electricity of the opposite sign is left in the cloud, the precipitation of the rain of one sign should result in an increase of electric stress, *i.e.*, potential gradient at the earth's surface; if, for example, *positive* electricity is being precipitated from a cloud, which is thus left electrically *negative* with respect to earth, then the potential gradient at the earth's surface should be positive and increase as long as the positive rain continues to fall. (As before, we understand a positive gradient as one which is represented by lines of force leaving the surface of the earth and ending on the cloud as in Fig. 9.) On the other hand, for example, if a cloud charged strongly negatively with respect to earth should precipitate its negative charge, then the positive potential gradient existing at the earth's surface should decrease.

We can say, therefore, that in general it is not possible to determine from the polarity of the charged rain the polarity of the cloud *taken as a whole* from which the rain fell, however, from the polarity of the rain and the direction of change of the potential gradient it is possible to deduce the polarity of the cloud.

However, in applying this principle in the field, it should be pointed out that we must determine the polarity of the rain falling over *an extensive area,* namely, the area covered by the cloud; a knowledge of the polarity of the rain falling *at a particular point* cannot be taken as indicative of the polarity of the cloud unless the rain falling at that point is representative of the rain falling over the entire cloud area. It would be necessary in the field then to erect several stations, distributed, say, a kilometre apart, in order to determine the true conditions within the cloud area. It would be extremely desirable in an experimental study of the thunderstorm to combine potential gradient meas-

urements, such as Norinder[3] has carried out, with rain current measurements, such as Simpson[4] has carried out, for three or more stations distributed over a circular kilometre or more. In this case we would expect a definite relation between the sign of the potential gradient, the sign of the rate of change of gradient, and the sign of the charge on the rain precipitated over the area (provided, of course, that the storm cloud does not move away from the area enclosed by the stations during the time of observation) as follows:

	Rain.	G.	dG/dt.	Cloud.	Stress.
(1)	+	+	+	−	Increasing
(2)	−	−	−	+	Increasing
(3)	+	−	+	+	Decreasing
(4)	−	+	−	−	Decreasing

Direction of Lightning Flash and Direction of Change in Gradient.—Similar difficulties, as those just discussed for the case of determining the relation between the sign of the charge on the *rain* and the polarity of the cloud, are encountered in the endeavor to determine, from the instantaneous change in the potential gradient at a given point produced by a lightning discharge, the direction of the lightning discharge, *i.e.*, whether the discharge was from a negative cloud to a positive earth or the reverse.

If all the lightning flashes were from cloud to earth, and if the point at which the change in gradient is observed were directly under the cloud, there would be no ambiguity. For if a negatively charged cloud strikes to earth, it is obvious that the gradient directly under the cloud just before the flash (Fig. 9) must have attained a large positive value (represented by lines of force leaving the earth) and have been reduced practically to zero by the flash; and this will be true also for a point on the earth at some distance from the centre of the area covered by the cloud, except that the gradient there will be much weaker.

However, in the case of a cloud-to-cloud flash (Fig. 2, for example), it is obvious that directly under the cloud (at *a*) the negative gradient existing immediately before the flash changes

[3] H. Norinder, *Tekniska Meddelanden f. Kungl. Vattenfallsstyrelsen*, Series E, No. 1 (1921).

[4] G. C. Simpson, *loc. cit.*

in the positive direction; while farther out (at *b*) the positive gradient existing before the flash changes in the negative direction. Similar relations would hold for the case represented by Fig. 3. From this it is obvious that the direction of the flash cannot be deduced from the instantaneous change in the gradient unless both the nature of the flash (*i.e.*, whether it was a cloud-to-cloud flash or a cloud-to-earth flash) and the location of the point of observation with respect to the flash are known.

Secondary Effects Produced by Primary Charges.—We have already touched on the production of secondary sparks by a cloud-to-cloud lightning flash, due to the release of previously "bound" *induced* charge. It is readily seen that with a heavy *primary* charge existing on a ground area, a cloud-to-earth flash must produce very powerful effects. When a lightning stroke occurs, unless a continuous conducting path is offered to the primary charges residing on the earth area, these charges will tend to "spark over" any non-conducting paths, *i.e.*, "gaps," in their endeavor to reunite with the opposite charge.

We can make this clear again from the transmission line case. The charge precipitated on the surface of the earth tends to creep upward toward the seat of the opposite charge (*i.e.*, the cloud) due to the mutual attraction between the two. The charge will consequently creep up on any elevated structures, in particular onto a transmission line. In this case the charge on the line is not an "induced" charge, it is a portion of the directly precipitated primary charge. Since the process of precipitation of charge takes appreciable time, this charge has sufficient time to creep over the high-resistance insulators onto the line. The charge held by the line will depend on its capacity to cloud, as we shall show quantitatively later. A sudden flash to earth produces a concentration of the charge of opposite polarity to that on the line directly under it, with a consequent sudden production of stress that the insulators cannot relieve sufficiently rapidly and "arc-over" occurs. We can describe the phenomenon by saying that that portion of the charge on the cloud which, previous to the lightning stroke, was "bound" by an equal and opposite charge on the line, is transferred during the flash from the cloud to the ground area directly under the line.

Quantitatively we can put, provided the line 2 is uncharged,

the overhead cloud 3 has a charge $-Q$, and the ground 1 a charge $+Q$:

$$+Q = a_{21} V_2 + a_{31} V_3 = Q_1$$
$$0 = a_{22} V_2 + a_{32} V_3 = Q_2$$
$$-Q = a_{23} V_2 + a_{33} V_3 = Q_3$$

whence:

$$V_2 = -(a_{32}/a_{22}) V_3 .$$

If under these conditions a lightning discharge takes place to ground, the line is practically unaffected, except for induced charges on the line reuniting, since after the flash we have $Q_1 = Q_3 = 0$, when also $V_2 = V_3 = 0$.

If, however, the line is connected to ground through a high resistance, we have before the flash for equilibrium: $V_2 = V_1 = 0$, and our equations become: $Q_1 = a_{31} V_3$, $Q_2 = a_{32} V_3$, $-Q = a_{33} V_3$, where $Q_1 + Q_2 = +Q$.

The instant after a lightning flash has passed, *i.e.*, after cloud 3 has been momentarily connected to earth, we have:

$$Q_2' = a_{22} V_2', \quad Q_1 = a_{21} V_2',$$

and, since the charge on the line is unchanged (perfect insulation), we have further $Q_2' = a_{22} V_2' = a_{32} V_3$, whence $V_2'/V_3 = a_{32}/a_{22}$, that is to say, the ratio of the potential of the line to ground immediately after a flash to the potential between cloud and earth before the flash, is equal to the ratio of the coefficient of induction of line to cloud and the coefficient of capacity of the line.

If there are only three bodies in the field, *i.e.*, if we can neglect the effect of other clouds, then we have $-a_{32} = a_{22} + a_{12}$, whence we have also $V_2'/V_3 = (1 + a_{12}/a_{22})$, which expresses the ratio of the potentials in terms of the coefficient of induction of line to ground and the coefficient of capacity of the line.

In terms of electric flux, we have for the flux φ_{32} from cloud to line before the flash the equation: $\varphi_{32} = a_{32} V_3 = Q_3 - Q_1 = Q_2$; and as the flux φ_{21} from line to ground after flash: $\varphi_{21} = a_{22} V_2' = Q_2$, which shows, as already stated above, that the voltage and stress between line and ground produced by a lightning flash is due to the transfer of the flux (tube of force) from cloud to line to line to ground, with consequent concentration of lines of force in the space between line and ground; it can be interpreted

also as a transfer of that portion of the charge in the cloud previously "bound" by the line, to the ground area immediately under the line. If the voltage so produced is applied sufficiently suddenly and is sufficiently large, an arc-over occurs from line to ground.

The "surge" on the line is the effect of the redistribution of the charge on the line. For example, if one end of the line is in a high field originally and the other in a low field, the first will carry a much larger density of charge than the other; when the charges are released by a lightning flash they will tend to distribute themselves evenly along the line and so a rush of charge from one portion of the line to the other will take place. Of course the rush of charge along the line is also accompanied by a rush of the opposite charge along the ground.

It should be emphasized that the equations above give only the relations in the initial and final states; the instantaneous voltages produced by the surge may be much higher than our calculated final voltage, since the latter is the voltage between line and ground when the charge is in equilibrium. It may be noted that even though a line is uncharged relative to earth, a surge may still occur on the line, but that this surge will not be likely to arc-over to earth. For although a line is not charged relatively to earth, a division of charges by induction may nevertheless occur on the line itself, one end receiving a positive, the other a negative charge, the algebraic sum of the two being zero. A lightning flash would "release" these charges, which would then rush to recombine and so produce a surge along the line.

Of course the relations just developed hold for any partially insulated body in the ground area, and is very likely the mechanism by which the majority of secondary sparks are produced by lightning flashes; it will be noted that these sparks can occur without lightning actually hitting the line or the insulated body. From another point of view we can regard these secondary sparks as part of the lightning discharge itself, since they really complete the neutralization of the charges.

Also, it may be pointed out, it is not necessary to have a high resistance from line to ground, the falling of rain on and the dripping of rain from the line would provide a mechanism by which charge can be put on the line.

IV. SUDDEN APPLICATION OF ELECTRIC STRESS.

Let us now turn to consider in greater detail a question already touched upon above, namely, whether it is possible suddenly to throw electric stress on an earth area where previously no stress existed.

Time Interval Required to Build up Field.—This would be possible, for example, if the process which charges the clouds relative to earth, *i.e.*, generates the voltage difference between cloud and earth, were almost instantaneous, *i.e.*, if the electric power developed by the thundercloud were very large. On Simpson's theory of the thunderstorm it is readily seen that this cannot be the case. It should take a very appreciable time to generate the potential difference between two clouds, and a still greater time to apply this potential difference between cloud and earth. To take a few examples, let us suppose that the velocity of the upward current of air in a thundercloud is 20 metres per second, it would then take 50 seconds for this current to cover 1 km., and it is likely that the space separation of the two charges must be much greater than this to affect the earth appreciably. Ordinarily we will have the base of the cloud at a height of about 1 km. above the earth, if now the negative charge is carried to a height of 2 km., we have only 1 km. between charges, with the result that the major portion of the stress must be from cloud to cloud as in Fig. 2. The same holds for a horizontal separation of oppositely charged clouds, it is not likely that this is effected with velocities much greater than 36 km. per hour, *i.e.*, 10 metres per second, and here again the clouds would need to separate several kilometres before the earth is appreciably affected. The separation of the clouds with the resulting change of cloud-to-cloud stress into cloud-to-earth stress (induction of charge at the earth's surface) is therefore theoretically also a process taking a very appreciable time. Lastly, if we consider the case where the charge, and consequently the electric stress, is transferred to earth by the *rain*, we have as the maximum velocity of falling rain in still air the value of 8 m. per sec., which again would require a time interval of the order of two minutes for the rain to fall to earth, *i.e.*, to traverse a kilometre. It is seen therefore that theoretically stress cannot be applied to earth very suddenly by any of the processes which generate the electrical charges and potentials.

In fact, this statement is amply supported by the experimental potential gradient curves of Norinder and C. T. R. Wilson [5]; these curves always show the gradient building up gradually with time (except for the effect produced by lightning flashes); they approximate curves given by the equation

$$G = G_\infty (1 - e^{-\beta t}),$$

the time of rise varying from 10 seconds in very extreme cases to several minutes, being usually, however, of the order of 50 seconds.

The "Impulsive Rush" Discharge.—It is then interesting to inquire, keeping in mind the transmission line case, whether a lightning flash itself can suddenly throw stress on an area of the earth's surface where, previous to the flash, no stress existed. These are the conditions of the "impulsive rush" lightning discharge of Lodge.[6] By the mechanism of the impulsive rush discharge, breakdown stress is supposed to be suddenly (practically instantaneously) thrown on an area of the earth's surface where, previous to the lightning flash, little or no stress existed. One condition under which this is supposed to occur is where there are two clouds one above the other, the upper discharges into the lower, and the lower to ground. In particular Lodge says, "It must sometimes happen when one cloud discharges into another that the potential of this other is suddenly raised high enough to discharge into the earth, even though no strain previously existed between it and earth."

Now it is true that stress can be suddenly thrown on an earth area by a lightning flash; *it is not likely, however, that sufficient stress to produce a breakdown (i.e., lightning) between a cloud and earth can be thrown suddenly on an earth area where previous to the flash no stress existed.* If the area underneath a storm cloud is already subjected to a stress near the breaking point, then it is theoretically possible for a lightning flash near this area to throw a sufficient *increment* of stress on the area to raise the stress to the breakdown point, *i.e.*, it is possible for one lightning flash to set off another; however, we never have a lightning flash

[5] C. T. R. Wilson, *Phil. Trans. Roy. Soc.*, A, **221**, p. 73, 1920–1921; see in particular Plate 4, Figs. 9 and 11.

[6] O. Lodge, "Lightning Conductors and Lightning Guards," p. 157; see also Figs. 23, 25, 27, in the same text.

suddenly occurring in an area where previously little or no stress existed.

We can easily dispose of the particular case cited. If an upper cloud sparks into a lower and thence to ground, it is obvious that the ground must have been the seat of the charge opposite to that on the cloud; as we have seen above in the majority of these cases the seat of the negative charge will be the upper cloud, the seat of the positive the ground, although the reverse may occur. Now it is to be noted that since the charging mechanism takes very appreciable time, the opposite charge must have been already present on the earth long before the flash, lines of force consequently left the earth and ended on the cloud, *i.e.*, stress existed between earth and cloud, long before the flash.

If clouds were good conductors, and parenthetically it may be pointed out that an additional and a very weighty objection against the impulsive rush phenomena is exactly the fact that clouds are not good conductors, then if we had two clouds one above the other, the upper being charged with a quantity of electricity $-Q$, the lower uncharged, and the earth with a charge $+Q$, we can write the following equations:

$$+Q = a_{21} V_2 + a_{31} V_3$$
$$0 = a_{22} V_2 + a_{32} V_3$$
$$-Q = a_{23} V_2 + a_{33} V_3$$

whence

$$V_2 = -(a_{32}/a_{22}) V_3,$$

that is to say, the potential of the lower cloud 2 would be intermediate between that of the upper cloud 3 and the earth; a fact which is also obvious from qualitative considerations, since the lines of force under these conditions must obviously leave the earth and end on the upper cloud, using the induced charges in the lower cloud as stepping stones. If the lower cloud were a good conductor, under these conditions, *i.e.*, $+Q$ on earth, $-Q$ on upper cloud, induced charges would appear on the intermediate cloud, a flash would occur then from the upper cloud to the lower and thence to ground, but it is doubtful whether such a flash, even if clouds were good conductors, would be essentially different whether the intermediate cloud were present or not.

If, on the other hand, we have the conditions $-Q$ on the upper cloud, $+Q$ on the lower, the flash would pass from cloud to cloud; there would be no flash to earth, nor would the earth be appreciably disturbed, except for the effect of very weak induced charges rushing together as in Fig. 5.

The earth is not to be regarded, as seems to have done frequently in the past, as a sort of reservoir of infinite capacity toward which electric charges are drawn; nor is a lightning flash to be regarded as a charged cloud *dividing* its charge with the earth; a lightning flash does not pass to earth unless the latter is the seat of the *opposite* charge to that in the cloud; and in any thunderstorm problem, in any electrostatic problem for that matter, we never deal with one charge alone (*i.e.*, we never deal with unipolar effects), we must always consider the effect of the presence of the other charge, that is to say, our conditions must always be subject to the relation: $\Sigma Q = 0$.

Doubtless the impulsive rush idea is suggested, to the present-day mind at least, by the transmission line case, where a lightning flash in the vicinity of the line suddenly produces a voltage between the line and ground, although, previous to the flash, line and ground were at the same potential.

For example, in a recent paper of Peek's [7] we find the statement, "the data so far discussed have been for voltages induced by cloud-to-cloud or cloud-to-ground discharge with a steady field between cloud and line at the instant previous to discharge. A different type of discharge is possible. There might be a cloud at or near ground potential directly above the line, while above this cloud there might be another at a very high potential. A discharge between these clouds could, in turn, cause an impulsive discharge between cloud and ground."

Now it is to be noted that if the upper cloud is at high potential, the only condition under which the lower cloud could be at or near ground potential is that it carry the major portion of the charge or sign opposite to that in the upper cloud, in which case it is true there would be very little field between the lower cloud and the earth previous to the flash; however, it is also true that the discharge would not be apt to pass to earth under these conditions but would take place only from cloud to cloud.

[7] F. W. Peek, JOUR. FRANK. INST., **199**, p. 161, 1925.

Quantitatively, if we index the upper cloud, the lower cloud, and the earth by 3, 2, and 1, respectively, we have before the flash according to hypothesis $V_3 >> 0$, $V_2 - V_1 = 0$, whence the charges are given by:

$$Q_3 = a_{33} V_3, \quad Q_2 = a_{32} V_3, \quad Q_1 = a_{31} V_3.$$

Now, since under these conditions a_{32} will be much larger in absolute value than a_{31}, we have Q_2 much greater in absolute value than Q_1, that is, the major portion of the flux would pass from the upper cloud to the lower, and the remainder from the upper cloud to ground. In terms of electric flux we have

$$\varphi_{32} = Q_2 = a_{32} V_3 \qquad \varphi_{31} = Q_1 = a_{31} V_3.$$

If now a lightning flash passes from the upper cloud to the lower, Q_2 is neutralized by an equal and opposite portion of the charge of the upper cloud, leaving only the excess, namely $(Q_3 - Q_2)$ to produce an electric stress between cloud and earth. In other words the flux φ_{32} vanishes, and the flux φ_{31} previously passing from upper cloud to earth now appears between lower cloud and earth, due, of course, to the surge of the charge $(Q_3 - Q_2) - Q_1$, into the lower cloud. As a result, if the clouds were good conductors, we would now have a charge $-Q_1$ in the lower cloud and a charge $+Q_1$ on earth, with consequently a potential V_2' between them given by $a_{22}V_2' - Q_1 - a_{31}V_3$.

Now as already stated, since a_{31} in this case is relatively small, it is doubtful whether V_2' would be sufficient to produce a discharge to ground, it would very likely only produce a sudden increment in the electric field existing before the flash.

In this connection it should be emphasized once more that the *whole* charge of the upper cloud previous to the flash does not appear in the lower cloud after the flash and so raise tremendously the electric stress between the lower cloud and earth; it is only the *excess* of charge which remains in the lower cloud after the cloud-to-cloud flash and which is effective in producing cloud-to-earth stress. Needless to say a lightning flash always effects a neutralization of charges.

It is seen from these examples that the impulsive rush requires rather special conditions: Arrangement of clouds, distribution of charges, etc., which would not be likely to occur often, even if it were possible for them to function afterwards, as Lodge believes.

Sudden Application of Stress.—Now while it is not likely that *breakdown* stress can be thrown by a lightning flash on an area in which previously there was little or no stress, it is true that sudden increments of stress can be produced by such means. However, these increments are not likely to be equal to the breakdown stress; although, in an area in which stress is already *near* the breakdown value, the increment might be sufficient to raise the existing stress to this value, and so cause a flash. In this case we would have one flash inducing another.

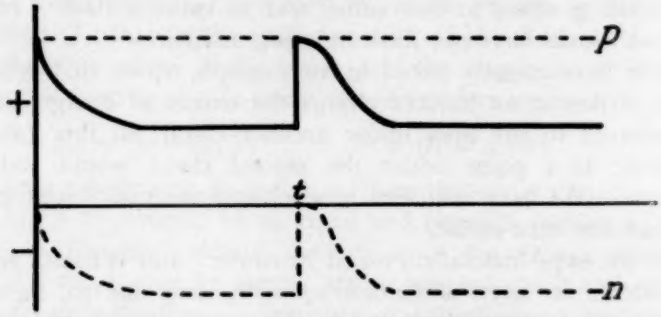
We have already noted in an example above that where a cloud strikes at an induced charge the excess of charge may be transferred to the area under another cloud; in this case the gradient at a point under the second cloud would suddenly increase. We have also seen how a cloud-to-cloud discharge can produce the same result.

In the experimental curves of Norinder [8] and Wilson,[9] sudden increments in stress simultaneous with, and due to, lightning flashes are frequently observed. However, the explanation of the majority of these increments, which are rarely of such a value as to induce breakdown, is quite different from any impulsive rush idea. The experimental gradient curves of Norinder and Wilson were usually taken at considerable distances from the storm itself, usually at least 1 km. and in most cases much farther than this (10–20 km.) from the storm centre. Now in a case where the observation point is at some distance from the seat of the storm and therefore the charges, the gradient at the point is due to the *superposition* of the effects of all the charges of the storm. It must frequently happen then that a positive gradient existing at a given point suddenly changes to a still larger positive value simultaneously with the discharge of a *negative* cloud, and *vice versa*. In this case the original gradient at the point was the *resultant* of a strong positive field and a somewhat weaker negative field; the actual value being the difference between them; the discharge of the negative, *i.e.*, the collapse of the negative, producing a sudden rise in the gradient due to the fact that the full positive field is now acting alone. With a number of storm clouds of both signs at various distances from the point of observation, all sorts of combinations are possible.

[8] H. Norinder, *loc. cit.*, Plates I–XIII.
[9] C. T. R. Wilson, *loc. cit.*, Plates 1–5.

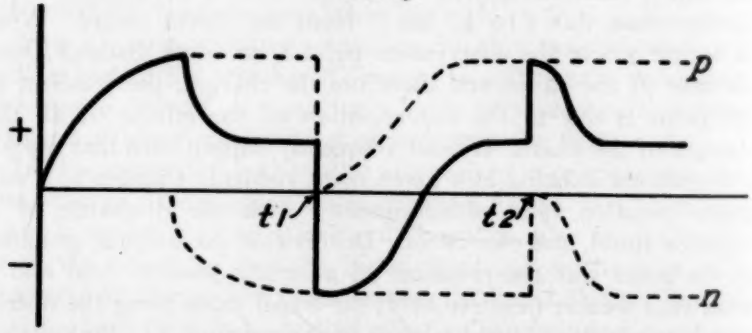
For example, the superposition of a steady positive field, p, due to a quiescent cloud, and a negative field, n, due to a charging and discharging cloud, can produce a resultant gradient given by the heavy line of Fig. 12. It will be noted that the discharge of the negative cloud by a lightning flash at the time t, results in a sudden increment of stress. Also the resultant field of two

FIG. 12.



actively charging and discharging clouds of opposite signs can produce a gradient varying as in Fig. 13. A very beautiful " saw tooth " curve of this type was obtained by C. T. R. Wilson.[10] In fact, curves of this type are frequently observed in the records

FIG. 13.



obtained by Wilson and Norinder. Another type of curve met with in the experimental records are some that look like the normal curve with the time axis reversed. These, it is readily seen, are produced by the superposition of two such curves as p and n of Fig. 13 in the interval t_2 to t_1. If the charges are prop-

[10] C. T. R. Wilson, *loc. cit.*, Plate 3, Fig. 8.

erly located with respect to a given point, the resultant field may be actually zero at that point, the discharge of either cloud producing in this case a sudden increment in the field at a point where previously no stress existed.

It is also to be noted that where these sudden increments occur in the curves of Norinder and Wilson, such increments are very rarely immediately followed by a complete collapse of the field, such as would occur if the sudden increment were followed by a breakdown, *i.e.*, a lightning flash.

But it must be noted that we would not expect a large percentage increase in the field existing directly under a storm cloud, *i.e.*, in a region of high electric stress. Under a storm cloud the effect of neighboring charges must always be negligible in comparison with the effect of the overhead charge, and while the effect might be present, it would produce only ripples in the already existing electric field.

V. NUMERICAL RELATIONS.

Gradients.—It is of interest to inquire what are the actual values of the potential gradients existing at the earth's surface during electrical storms. Obviously the gradient over a smooth level area always must be less than the dielectric strength of air, since the field existing between cloud and earth is not uniform. However, locally gradients of the order of the dielectric strength of air, *i.e.*, 30,000 volts/cm., may nevertheless exist, even though the gradient over a level surface (gradient over the flat) is far below the dielectric strength of air. In particular within a limited region at the top of a pointed elevated rod (a lightning rod, for example) a gradient of 30,000 volts/cm. can easily exist and silent discharge take place, even though the actual gradient over the flat and at points outside of the zone at the top of the rod are far below this value. We might even expect brush discharges to occur, and in fact these are observed in nature in the form of St. Elmo's fire,[11] and from this fact we may conclude at once that locally (*i.e.*, in small zones) gradients of the order of 30,000 volts/cm. do occur. It does not follow that lightning necessarily must take place when the gradient at any point reaches a value equal to the dielectric strength of air, unless the

[11] For an excellent detailed discussion of the phenomenon of St. Elmo's Fire, see A. Gockel, "Das Gewitter," pp. 79-97 (1925).

tube of force reaching from this point to the cloud is of such a shape that the field is very nearly uniform throughout. If the cross-section of the tube increases rapidly, the zone in which the field is sufficiently intense to produce ionization by collision is limited to a zone near one end of the tube, ions outside this zone cannot travel with sufficient velocity to sustain the process of ionization by collision, *i.e.*, produce a lightning flash.

The potential gradients actually existing *over level surfaces* during storms have been measured by Wilson and Norinder. Wilson observed no values above 300 volts/cm., however, nearly all of his observations were taken at some distance, usually 5–10 km. from the storm. Norinder finds from his observations that gradients of 1000 volts/cm. are common during electrical storms, that gradients of the order of 2000 volts/cm. also occur, and surmises that near the path of the lightning flash (and previous to it) gradients of the order of 3000–4000 volts/cm. exist. Norinder's observations were made to within 1 km. of the storm centre.

If we can extrapolate Peek's needle gap curves to distances of 1 km. or more, if a lightning discharge obeys the needle gap curve, and if the electric field over the flat, *i.e.*, potential gradient over the flat, can be taken as equal to the voltage divided by the distance, then we would obtain a maximum gradient of 3300 volts/cm. as existing during electrical storms.[12]

Charges.—Let us next consider the density of charge corresponding to the observed gradients over the flat. This is readily obtained from the relation

$$\delta = G/4\pi .$$

If we take 3300 volts/cm. as the maximum gradient existing before the flash, this would correspond to a maximum charge of *2.3 microcoulombs per circular metre* (area of circle 1 metre in diameter), and 2.3 coulombs per circular kilometre provided this gradient is maintained over the entire area of a circular kilometre. Now the remarkable fact about the surface density, and one which has an important bearing on the problem of lightning *prevention,* is the relatively *small quantity* of *electricity* stored on even an extensive area during an electrical storm—it is exactly this fact which makes it seem entirely feasible to dis-

[12] F. W. Peek, JOUR. FRANK. INST., 199, p. 142, 1925.

charge a storm cloud and actually to prevent the lightning flash from occurring. Of course when we speak of discharging the cloud we need to take into account also the time factor; in other words, we must consider not only the amount of *charge* stored up but also the *time* during which the charge builds up, in particular the ratio dQ/dt, *i.e.*, the *current*, since any problem of lightning prevention will involve a balancing of the downward convection current produced by the charged rain by an equal upward current.

As already stated above, aside from sudden increments produced by lightning flashes, the gradient at any place on the earth's surface during a storm builds up gradually—from a few simple calculations above we arrived at a time interval of the order of 50 sec. If we take a value of 50 sec. as the required time interval—this value is also amply supported by the field data of Norinder and Wilson—we find that for a value of the gradient of 3300 volts/cm., *i.e.*, a charge of 2.3 coulombs per circular km., this corresponds to an equivalent current of 46 milliamperes per circular km. Actually, of course, the charge builds up according to the equation $G = G_\infty\ (1 - e^{-\alpha t})$ the instantaneous current gradually decreasing, *i.e.*, dG/dt being given by:

$$\frac{dG}{dt} = \alpha G_\infty e^{-\alpha t}.$$

It is of interest to compare this calculated value with the actually measured value of the convection current produced by falling rain. An extensive series of such measurements has been carried out by Simpson,[13] who observed that during the average electrical storm the rain carries down an amount of charge per second equivalent to 300×10^{-15} amp./cm., in one exceptional storm a current of 990×10^{-15} amp./cm. was recorded. These values would correspond, respectively, to a downward current of 2.3 and 7.6 milliamperes per circular kilometre, and, it is to be noted, are only .05 and .17, respectively, of the 46 milliamperes calculated on the basis of $G = 3300$ volts/cm. and $t = 50$ sec. In fact, if we use Simpson's values for the convection current per cm.^{2}, we have as the required time interval to build up a charge of 2.3 coulombs per circular km., values of 1000 sec. and 300

[13] G. C. Simpson, *loc. cit.*

sec., respectively—a much longer time interval than is actually observed from the experimental curves of Norinder and Wilson.

This discrepancy remains at the present time unexplained. However, we may indicate the following possibilities: (1) The currents observed by Simpson may be too low. In this connection it may be noted that Simpson obtained the current by observing the charge accumulating in an insulated vessel during *two-minute intervals;* it is obvious that he obtained in this way only the *average* current during the interval; if the instantaneous current should rapidly change from positive to negative values, Simpson's method would give too low a value.

Since the gradient is observed to change from zero to a maximum in as short a period as 10 sec., it is obvious that the intervals to be chosen in rain current measurements should also be of this order, namely, 10 sec., rather than 120 sec. as Simpson used.

It would be desirable to repeat Simpson's measurements with smaller time intervals, or better, if possible, by observing the instantaneous currents. The fact that both positive and negative drops intermingle [14] need not concern us for this purpose, since we are interested only in the average instantaneous current, and it is only the algebraic sum which remains after the precipitation has reached the earth and is effective in producing electric stress. Another possibility is that at no time was Simpson directly under the storm centre when he made his measurements, *i.e.*, that much larger values of the current than 990×10^{-15} amp./cm. actually occur in the region near the lightning flash.

(2) The area over which precipitation occurs and the area on which the charge subsequently concentrates do not coincide. For example, it may be that the charge after being precipitated over a relatively large area should draw in and concentrate in a smaller area; if this is so then to produce the same surface density within the smaller area we would require a much smaller rain current density than if the charge actually remains in the area over which it has fallen. However, it seems likely that the reverse actually occurs, namely, that the precipitated charge actually spreads out, rather than concentrates, after precipitation. In this connection it is important to point out that the charge which has fallen to earth does not diffuse over the whole earth

[14] P. Gschwend, *Jahrb. d. Rad. u. Elek.*, 17, p. 62, 1921.

and so practically disappear; the precipitated charge is still "bound" by the charge of opposite polarity in the cloud. It may nevertheless spread locally. So far little is known of the distribution of charge under a storm cloud, that is, no one has as yet mapped the electrostatic field of a storm cloud. Norinder [15] has determined the approximate extent of the field, namely, that it extends to at least 1 km. and not more than 10 km. from the storm centre. The manner in which the field falls off with distance has also been approximately studied by plotting the instantaneous change in gradient produced by a flash against the observed distance to the point of discharge.

If the change in gradient ΔG at the earth's surface were everywhere known, the charge involved in a lightning flash could of course be directly calculated from the equation:

$$Q = \frac{1}{4\pi}\int \Delta G ds .$$

(3) The assumed time interval of 50 sec. is too low. In the experimental curves of Norinder and Wilson no distinction is made between cloud-to-cloud and cloud-to-earth discharges. We might expect, however, that a cloud-to-cloud field could be built up much more rapidly than a cloud-to-earth field. When observations are made at some distance from the storm, of course both types of fields are of the same order of magnitude. It would be interesting to examine the gradients directly under a storm cloud to see whether the charging interval in this case is of longer duration.

[15] H. Norinder, *Electrical World*, **83**, p. 223, 1924.

Canadian Hydro-electric Power Development. BRYSSON CUNNINGHAM. (*Nature*, Aug. 27, 1927.)—Hydro-electric development is much more important in Canada than in Great Britain, not only because of the small amount of this type of power available in the British Isles and of the presence of great supplies of coal, but also because in Canada the two most populous provinces, Quebec and Ontario, present the most striking contrast between the riches of their water power and the poverty of their coal fields. Of the 18¼ million horsepower available from water power in all of Canada about two-thirds is in these two provinces, while practically no coal is found within their borders. In the estimate just cited the total contrasts vividly with the one million h.p. given for the British Isles, yet the Canadian figure does not include streams in the north that will later literally be put on the map. The author ventures the prediction that eventually the total will reach 40,000,000 h.p. At the present time 4,550,000 h.p. is actually developed, 10 per cent. of which is in British Columbia, 39 per cent. in Ontario and 42 per cent. in Quebec. According to the data issued by the Water Powers Branch of the Canadian Department of the Interior, Saskatchewan with possibilities of 513,000 h.p. develops at the present time no more than 35. Each installed horsepower is counted as replacing six tons of coal per annum.

Of the total horsepower obtained hydro-electrically in Canada, 81 per cent. is developed in central electric stations, 12 per cent. in paper and pulp mills and the remainder in other industries. For the whole Dominion with its population of 9,390,300 on June 1, 1926, the installation per 1000 inhabitants is 485. Yukon and Northwest Territory leads in this ratio with 1073 h.p. per 1000, but this is due to the smallness of its population, which is only 12,300, rather than to the imposing extent of its hydro-electric plants. In Ontario and Quebec the ratios are, respectively, 569 and 748 h.p. per 1000. In 1926 266,000 was added to the sum of Canadian installed hydro-electric horsepower and the projects under way will contribute 1,700,000 more to the already impressive total, representing investments running into the hundred millions of dollars. For the financing of new installations there is little difficulty in securing the requisite capital because plants already in operation have shown consistent earnings and their output has been absorbed with promptness.

G. F. S.

Sprint Running.—In a study made at Cornell University, K. FURUSAWA, A. V. HILL, and J. L. PARKINSON (*Proc. Royal Soc.*, B, 1927, **102**, 43–50) found that the mechanical efficiency during sprint running was about 38 per cent. During a 200-yard dash, the maximum velocity attained was 11.46 yards per second with an energy output of 8.5 horsepower; the runner was then producing over 4 grams of lactic acid per second in his muscles.

J. S. H.

NOTE ON A TENTATIVE HYPOTHESIS OF THE LATENT IMAGE.*

BY

A. P. H. TRIVELLI.

IN ORDER to explain the higher light sensitivity of the larger silver bromide grains in one and the same photographic emulsion, Sheppard, Trivelli and Loveland [1] proposed a so-called concentration speck theory, which later was somewhat modified to agree with newly discovered facts. According to this hypothesis the action of the silver sulphide centre [2] " is confined to increasing its size by accretion of photochemically reduced silver atoms to form a nucleus large enough to induce developability. It is evident that the larger the speck is, below a certain limit giving spontaneous developability, the less the added number of silver atoms necessary to make the grain developable. Hence the less the exposure required for this and the greater the apparent sensitivity of the grain." The presence of silver sulphide specks (silver may also serve as a centre [3]) involves a strain in the silver halide lattice, as already suggested by Sheppard and the writer [4] in the case of silver iodide crystallized with silver bromide. This strain in the crystal lattice, and particularly in the immediate neighborhood of the silver sulphide sensitizing speck, corresponds to a certain disorientation of the lattice and to a deformation of the ions of the lattice in the region extending out from the speck, the deformation becoming less as we get away from the speck. The energy absorption which takes place in the neighborhood of a silver sulphide speck probably occurs in

* Communication No. 313 from the Research Laboratory of the Eastman Kodak Company.

[1] S. E. Sheppard, A. P. H. Trivelli and R. P. Loveland, JOUR. FRANK. INST., **200**, 51 (1925). S. E. Sheppard and A. P. H. Trivelli, to appear shortly in *Phot. Korr.*

[2] S. E. Sheppard, *Coll. Symp. Monogr.*, **3**, 76 (1925); *Phot. J.*, **65**, 380 (1925).

[3] E. P. Wightman and R. F. Quirk, paper to be published shortly.

[4] A. P. H. Trivelli and S. E. Sheppard, "The Silver Bromide Grain," 121 (1921).

discrete quanta.[5] We regard the photochemical reduction of silver halide as consisting fundamentally in the transfer of a valency electron from a halide ion to a silver ion of the crystal lattice.[6] The electron orbit, whose deformation would be of principal importance in this view, is that of the valency electron of the bromide ion. We suppose that immediately contiguous to a silver sulphide speck the perturbation of this orbit is highest, falling off as we move away from the speck. When light falls on a grain containing a sensitizing speck, there is a transfer of energy toward the speck, resulting in reduction of silver around the sensitizing speck.

It should be noted that the conditions are strongly favorable to a decomposition at the boundary of the "speck" and silver halide, both optically and chemically. There is a discontinuity in refractive index at the boundary, while it is known that in heterogeneous catalysis the reaction takes place at the boundary of the two phases.[7] The hypothesis, that the sensitizing silver sulphide speck operates to concentrate the photochemical product about it, and to initiate development, when it reaches a certain size appears to be definite. Microscopic investigations on the visible photochemical decomposition of pure silver bromide crystals of a photographic emulsion, on which sensitizing specks of silver sulphide of a microscopically visible size were produced,[8] showed, at 2500 × magnification, that around the speck silver was formed in the metallic state. On other places on the crystal surface photohalide appeared in small spots in blue and blue-green colors. This can be regarded as evidence that around the silver sulphide speck the photochemical decomposition is much more compact and hence more complete than in other places. Besides this direct addition of silver Hickman[9] has suggested

[5] G. B. Gudden and R. Pohl, *Physik. Zeitschr.*, **21**, 529 (1921). J. Eggert and W. Noddack, *Die Naturwiss.*, **15**, 57 (1927), who have given a very full bibliography of the subject.

[6] *Cf.* S. E. Sheppard and A. P. H. Trivelli, *Phot. J.*, **61**, 303 (1921). Independently proposed by R. Fajans, *Zeitschr. Elektrochem.*, **28**, 499 (1922).

[7] *Cf.* H. S. Taylor, "A Treatise on Physical Chemistry," **2**, 952 (1925).

[8] S. E. Sheppard, A. P. H Trivelli and E. P. Wightman, "The Production of Sensitizing Specks on Silver Halide Grains," *Phot. J.*, **67**, 281 (1927).

[9] K. C. D. Hickman, *Phot. J.*, **67**, 34 (1927). *Cf.* also R. Lambert and E. P. Wightman, paper discussing Hickman's hypothesis, *J. Phys. Chem.*, **31**, 1249 (1927).

that the silver sulphide serves as an absorber of the liberated halogen and in so doing gives rise to more silver than that produced by photochemical decomposition of the silver bromide. There is no conflict between Hickman's hypothesis and that proposed above.

The mechanism proposed for the orientation factor is difficult to handle quantitatively. I now propose a further hypothesis with regard to the energy transfer from the place of absorption to the silver sulphide speck, which seems to have more possibility of opening up for quantitative investigations a part of the mechanism of latent image formation.

In 1868, a photo-electric phenomenon was described by Becquerel.[10] A platinum plate was covered with the chloride, bromide or iodide of silver. When this plate was dipped in acidified water or in a solution of the corresponding alkali halide, it was observed that the potential of the platinum against the liquid was increased by illumination. In the light the plate was more positive than in the dark. Becquerel discovered also that the effect is less with metallic silver than with platinum.

If we transfer this photo-electric phenomenon to the silver sulphide centre in the silver halide crystal, whereby the silver sulphide takes the place of the platinum, then one should expect an increased difference of potential between the silver sulphide and the surrounding silver bromide during illumination, which difference of potential is discontinuous in the silver bromide, as will now be explained.

The silver bromide crystal shows, as is well known, high adsorptive power for foreign substances, and when it grows in contact with these substances, they become distributed more or less regularly but discontinuously through it. W. Reinders[11] has shown, for example, that in silver bromide crystals the gelatin is distributed in small molecular complexes through the entire crystal. Thus the silver bromide crystal is analogous in structure to Lenard's inhomogeneous crystal phosphorogen with its molecular complexes of foreign substances, which he called "centres"[12] and which are the places at which electrons are

[10] E. Becquerel, *La Lumière*, **2**, 121 (1868).
[11] W. Reinders, *Zeitschr. phys. Chem.*, **77**, 213, 356, 677 (1911).
[12] P. Lenard, *Zeitschr. Phys.*, **3**, 98 (1920); **4**, 206 (1921).

liberated. Lenard and Saeland [13] have made an extensive examination of the photo-electric properties of these substances. During illumination the surface charges up positively. It was found that when the surface ceases to emit any more electrons, only 1/10 of the surface is really charged. This justifies the authors in considering that the photo-electric effect is not distributed over the total surface but is localized in those "centres" which are active in a phosphorescent sense. By using colored glasses as light filters it was established that the light, which did not produce phosphorescence, did not produce the photo-electric effect. The view given is that electrons leave the "centres" and the phosphorescence is produced by the electrons falling back into the charged centres in the dark. Although it is quite probable that there is a liberation of electrons at the interface between the centres and the surrounding silver bromide, there are cases known where electrons are emitted from places where we hardly can accept the existence of impurities in the crystal. For instance in diamond, impurities detectable very easily by their absorption spectra diminish the amount of the photo-electric current about 10 to 20 per cent.[14] The same is true for zinc sulphide crystals under certain conditions. But under other conditions, when the impurities are introduced in some other way, diamond and zinc sulphide show strong phosphorescence. That is, the method of the introduction of the impurities is the all-important factor in enhancing or decreasing phosphorescence and the photo-electric properties. Similarly, optical sensitizing of silver bromide is very dependent in effect on the technique employed. It has been shown by Leszynski [15] that sensitizing dyes as normally used do not cover the entire surface of the silver halide grains, in other words, they exist in spots on or near the surface. It has also been shown by E. R. Bullock [16] that when silver sulphide is formed in the silver halide grain in a quite different way than that of chemical sensitizing through allylthiourea, *i.e.*, with diluted thiosulphate solution, a totally different optical sensitizing effect is produced, which could be explained by the fact that here another modification of silver sulphide is formed than with allylthiourea. Wightman,

[13] P. Lenard and S. Saeland, *Ann. d. Phys.*, **28**, 476 (1909).
[14] B. Gudden and R. Pohl, *Zeitschr. Phys.*, **3**, 123 (1920).
[15] W. Leszynski, *Zeitschr. wiss. Phot.*, **24**, 261 (1926).
[16] *Cf.* S. E. Sheppard, *B. J.*, **73**, 33 (1926).

Trivelli and Sheppard [17] have shown that photohalide emulsions prepared by different methods give optically different photographic sensitizing. It seems also reasonable to suppose that the silver halide grain resembles in structure the Lenard phosphorogens. In both cases not only is the nature of the impurity, but also the way the impurity is introduced into the crystal lattice, of great importance in connection with the liberation of photoelectrons. Among the various kinds of impurities in the silver halide crystals of the highly sensitive photographic emulsions, the silver sulphide specks are of special interest.

It is assumed that upon illumination a number of elementary electric currents will appear between the silver sulphide speck and the charge discontinuously distributed in the surrounding silver bromide lattice,[18] where foreign substances are imbedded, due to the potential difference produced. These currents may spread out from the silver sulphide speck towards the different spots in the surrounding silver bromide.

Toy and Edgerton [19] have shown that at wave-lengths less than $\lambda = 280m\mu$ silver sulphide prepared by fuming metallic silver with hydrogen sulphide, is from two to three times as active photo-electrically as silver iodide prepared in a similar way and about twenty-five times more active than silver bromide. Traces of silver sulphide in silver bromide have no abnormal sensitizing effect photo-electrically for the method of preparation used. No mention is made of a differentiation into two modifications of silver sulphide. We do not know, as yet, which of the two known forms gives the sensitizing effect; or, if they both give sensitizing effects, what the difference is, if any. But if we assume that Toy and Edgerton have introduced the same modification of silver sulphide into the silver bromide as is formed by the alkaline treatment of the double compound of silver bromide with allylthiourea, then this photo-electric effect may still be a factor in the formation of the latent image, in spite of the fact that they did not find a parallelism between the photo-electric

[17] E. P. Wightman, A. P. H. Trivelli and S. E. Sheppard, "Preparation and Properties of Experimental Photohalide Emulsions," paper to be published shortly.

[18] The absorption of radiant energy is supposed to take place according to the laws of chance.

[19] F. C. Toy and H. A. Edgerton, *Phil. Mag.*, [7] **3**, 482 (1927).

properties of pure silver halides and the photographic properties of silver halide emulsions. This photo-electric effect may be partly the cause of the change in potential differences in and on the surface of the silver bromide grain during illumination, and the existence of photo-electric currents in the silver halides through illumination is likewise possible.[20] We do not know, however, exactly how they run. It is quite probable that the solution of the problem of the latent image, with its complexity of reversal phenomena, depends on the knowledge of these elementary photo-electric currents. With our present state of knowledge, about all we can say is that very probably the silver sulphide speck during illumination gives off electrons to the surrounding silver bromide. But such a process would lead very quickly to an equilibrium condition through the increased positive charge of the silver sulphide speck. In any event, such a current would explain beautifully the deposition of silver atoms at the boundary of the silver sulphide speck and the silver bromide crystal lattice, if we only knew how the electrons return back to the speck. In that case we could regard the orientation action of the concentration speck as that of an electrode (cathode) in electrolysis.[21] The electrolysis of the crystallized silver halides and their mixtures has been studied by a number of investigators. Kohlrausch [22] and Lehmann [23] state that an electric current through solid cubical silver iodide gives electrolytic depositions of silver and iodine. Kohlrausch also investigated the conductivity of silver bromide and silver chloride. Lorenz and Czepinski [24] were able to show the existence of measurable polarization currents at temperatures from the melting-point down to 320° C. Tubandt and E. and F. Lorenz [25] have investigated the electrolysis of the crystallized silver halides and their mixtures, and they were able to show that in all cases Faraday's law holds.

[20] *Cf.* C. Ries, "Das Licht in seinen elektrischen und magnetischen Wirkungen" (1909).

[21] The complexity of the photo-electric phenomena is well described in A. L. Hughes' "Photo-electricity" (1914), and W. Hallwach's "Die Lichtelektricitat," "Marx's Handbuch der Radiologie," Vol. III (1916).

[22] W. Kohlrausch, *Ann. d. Phys.*, **17**, 642 (1882).

[23] O. Lehmann, *Ann. d. Phys.*, **24**, 1 (1885).

[24] R. Lorenz and V. Czepinski, *Zeitsch. anorg. Chem.*, **19**, 208 (1909).

[25] C. Tubandt and E. Lorenz, *Zeitsch. phys. Chem.*, **87**, 513 (1914); C. Tubandt and F. Lorenz, *Zeitsch. phys. Chem.*, **87**, 543 (1914).

Attention is also called to the investigations of Benrath and Wainoff [26] on the electrolysis of solid silver chloride.

Lehmann [27] states that microscopical investigations of the electrolysis of cubical silver iodide showed that only the silver ions in the crystal lattice move, while the anions remain immovable. Tubandt, Eggert and Schibbe [28] have continued these investigations with the silver halides and $\alpha-$ and $\beta-$ silver sulphide and confirm Lehmann. Silver sulphide in the $\beta-$ modification, stable below 179° C., showed a more complicated conductivity than the $\alpha-$modification. Hexagonal silver iodide has much less conductivity than the regular silver iodide with the diamond lattice. They found that at the cathode, metallic silver is formed in the form of fine threads and dendrites, which had been already observed by LeBlanc and Kirschbaum [29] in silver chloride and by Hittorf [30] in silver sulphide. Tammann [31] has found that silver iodide is transformed at high pressures (3000 atm.) into a cubical kind of crystal, which coexists with the hexagonal to a great extent. Barth and Lunde [32] made an exhaustive study of mixtures of silver bromide and silver iodide. They also found on X-ray studies that silver iodide may go into the silver bromide lattice if its concentration is not too high, which was already known through previous crystal-structure studies of R. Wilsey of this laboratory.[33] They were able to show that Tammann's modification of silver iodide is in all probability cubical with the silver bromide structure. Now Graetz [34] found that the conductivity of silver iodide at pressures at which Tammann's modification is formed is about 200 times greater than the regular iodide. Therefore we may expect in the silver bromide-silver iodide mixed crystals at low concentrations of silver iodide a great increase of conductivity with increasing

[26] A. Benrath and J. Wainoff, *Zeitsch. phys. Chem.*, **77**, 257 (1911).

[27] O. Lehmann, *Ann. d. Phys.*, **24**, 1 (1885).

[28] C. Tubandt and P. Eggert, *Zeitsch. anorg. Chem.*, **110**, 196 (1920). C. Tubandt, *Zeitsch. anorg. Chem.*, **115**, 105 (1920). C. Tubandt, S. Eggert and G. Schibbe, *Zeitsch. anorg. Chem.*, **117**, 1, 48 (1921).

[29] M. LeBlanc and F. Kirschbaum, *Zeitschr. Elektrochem.*, **16**, 242 (1910).

[30] Hittorf, *Pogg. Ann.*, **84**, 1 (1851).

[31] C. Tammann, *Zeitschr. phys. Chem.*, **75**, 733 (1911).

[32] T. Barth and G. Lunde, *Norsk. Geol. Tidsk.*, **8**, 293 (1925).

[33] R. B. Wilsey, JOUR. FRANK. INST., **200**, 739 (1925).

[34] Graetz, *Ann. d. Phys.*, 29, 314 (1886).

content of iodide. Still the investigations of Tubandt and his collaborators did not show any remarkable increase of conductivity if the silver bromide contains small amounts of silver iodide. But all the conductivity measurements mentioned above were made at different high temperatures near the melting-point on account of the very low conductivity of the silver halides at normal temperatures. This seems a great objection to the possibility of an elementary photo-electric current in the silver halides during illumination, causing the formation of the latent image, if the silver halides have not a quite different electrical conductivity in the light from that which exists in the dark.

The investigations of Arrhenius [35] have shown that the electrical conductivity of the chloride, bromide and iodide of silver on glass plates in thin, dry layers between silver electrodes is markedly increased when illuminated, especially with blue and violet radiation. He regarded this property as a photo-conductivity similar to that in selenium. Scholl [36] repeated this experiment on silver iodide and obtained in the light at $\lambda = 430\mu\mu$ a conductivity forty to fifty times greater than in the dark. With Bädeker [37] he regarded this increased conductivity as due to the formation of metallic silver in the light. In the dark the reversed reaction should reëstablish a great part of the resistance. Wilson [38] studied simultaneously the photo-electric effect and the increase in conductivity due to illumination in silver iodide. The change in conductivity was produced, both by violet and ultra-violet light, the effect due to the latter being only 8 to 10 per cent. of that due to the former. The emission of photo-electrons occurred only when ultra-violet light was used. Wilson suggested the following important explanation, based on a number of experiments. Violet light causes the atoms to emit electrons with small velocities. These are easily directed by the applied electromotive force to increase the conductivity. Ultra-violet light releases electrons of higher speed, which are less easily directed by the electromotive force, and hence the change in conductivity is less. In other words, the slow photo-electrons are more easily converted into free electrons, which determine

[35] S. Arrhenius, *Sits. ber. Akad. Wiss. Wien.*, **96**, 831 (1887).
[36] H. Scholl, *Ann. d. Phys.*, **14**, 41, 193 (1905).
[37] R. Bädeker, *Ann. d. Phys.*, **29**, 566 (1909).
[38] W. Wilson, *Ann. d. Phys.*, **23**, 107 (1907).

the conductivity, than the faster photo-electrons. The facts are unfortunately still more complicated, as is shown by later investigations of Gudden and Pohl.[39] These investigators came to the following general conclusions in regard to photo-conductivity:

(1) The photo-electric current increases in the beginning and decreases at the end of an illumination. Fig. 1 shows such a curve. This curve was obtained with cadmium sulphide, which has a very great "inertia." A similar curve for silver iodide is published by Scholl[40] and shows also considerable "inertia."

(2) The photo-electric current increases less than proportionally with the light intensity. Different curves are obtained according to different wave-lengths and the potential applied.

(3) Different kinds of relations are obtained between the photo-electric current and the applied potential, depending upon the light intensity and the wave-length.

(4) For constant light energy and constant potential the photo-electric current is dependent on the wave-length of the light.

(5) The distribution of spectral sensitivity changes with longer action of certain wave-lengths. Previous maxima disappear and new ones appear.

Gudden and Pohl succeeded, by using very weak illumination and short exposures, in decomposing the photo-electric current into two components: *A primary current, which starts instantaneously and which reaches a maximum and is proportional to the absorbed energy, and an electrolytic secondary current, which increases in the beginning proportionally to the time of illumination.*

The intermittency effect of the photographic plate and other phenomena show clearly that the photographic effect is not an instantaneous one. In other words, the instantaneous photo-electric effect is insufficient for a photo-electric theory of the latent image. This means that for the latent image formation attention must be focussed on the electrolytic secondary current. This current increases with the potential and the energy density of the light and is the cause of the great complexity of phenomena of photo-conductivity.

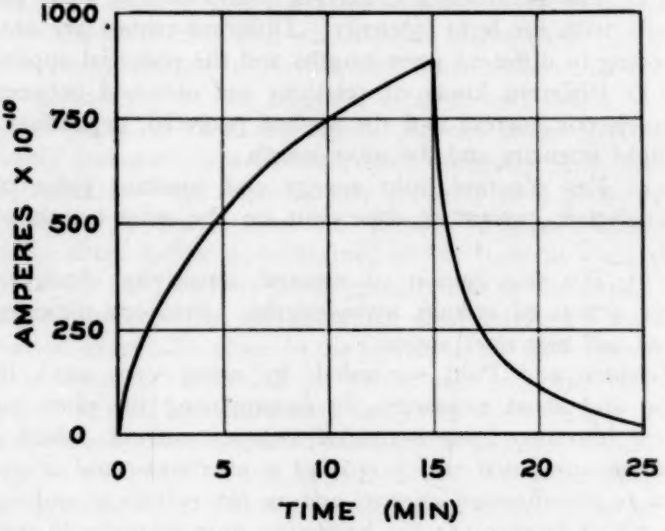
[39] B. Gudden and R. Pohl, *Zeitschr. Physik.*, **6**, 248 (1921); **7**, 65 (1921), *Physik. Zeitschr.*, **22**, 529 (1921).

[40] H. Scholl, *Ann. d. Physik.*, **14**, 193, 417 (1905).

If we look at Figs. 1 and 2 and derive from the curves the consequences for an intermittent exposure, we get with increasing intermittence a smaller photo-electric current, which will decrease to a limit, when the conductivity is so much decreased that it reaches the value of the dark conductivity. Correlation between these two phenomena must be determined experimentally.

It is quite possible that in this way we shall be able to account for the failure of the Bunsen-Roscoe reciprocity law. It is very

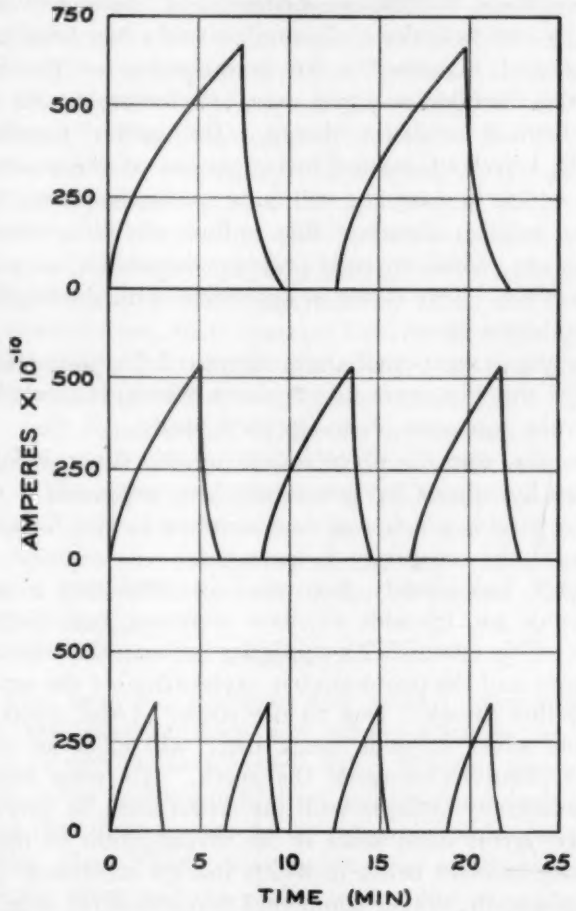
FIG. 1.



easy to see that with low intensity the electrical resistance increases, so that the photochemical effect becomes smaller. With increasing light intensity the photochemical effect increases. Mr. V. C. Hall, of this laboratory, has suggested that it may even be possible to obtain a correlation between conductivity and the maximum of blackening at certain intensities and certain time exposures. With low intensity the maximum of electrical conductivity in photo-conductive materials is much more quickly reached than with higher intensities at which it requires a longer duration of time according to the nature of the substance. In a study of the reciprocity law It is held constant in the determination of the density of the photographic plate. Therefore, for increasing values of I the time t always becomes proportionately

shorter and shorter so that at the intensity, when t begins to become less than the time of exposure necessary to reach maximum conductivity, then the attained maximum of the photo-

FIG. 2.



electric current (and hence of the amount of photographic density) will become less.

Sheppard[41] has already in 1924 indicated a connection

[41] S. E. Sheppard, paper on "Antagonism of Radiations in Photochemical and Photographic Reactions," read at the Washington meeting of Am. Chem. Soc., 1924, which will be re-written and published in the *Chemical Reviews*.

between the Herschel effect and the extinction of phosphorescence in the extreme red and the infra-red light. Dr. E. P. Wightman, of this laboratory, has also independently pointed out to me this relation.[42] In this connection, attention must be paid to the actino-dielectrical current of Lenard and Saeland,[43] which is induced by red radiation. A similar study has been made by Ramsauer and Hausser.[44] An investigation of the Herschel effect in this direction may give more satisfactory results than the present chemical oxidation theory. The author, together with Mr. R. P. Loveland, started investigations of the reversal phenomena, which in part are still in a qualitative state, but they show that halogen absorbers like sodium nitrite, acetone, semicarbazone, etc., which strongly prevent solarization, accelerate the Herschel effect. This is not in agreement with the requirements of the oxidation theory.

In the paper mentioned above Sheppard discusses also photoelectrically the Becquerel effect about which H. Scholl[45] has made a very extensive photo-electrical study.

As to the rôle of silver iodide in the silver iodobromide crystals an hypothesis has previously been suggested,[46] that due to a lattice strain, which was demonstrated in the X-ray studies of Wilsey,[47] the sensitivity is increased.

Graetz[48] had already discovered in 1886 that a strain in solid chloride and bromide of silver increases their electric conductivity. This establishes a similarity between the concentration speck theory and the photo-electric explanation of the orientating action of this theory. Due to the strain of the crystal lattice around the silver sulphide speck there will exist an increased electric conductivity towards the speck. To what extent the photo-conductivity changes with the strain must be investigated.

Of very great importance is the investigation of the photoconductivity in silver halide in which foreign substances (gelatin, silver sulphide, the double compound thiourea-silver sulphide, sil-

[42] *Cf.* P. Auger, *Ann. Phys.*, **6**, 183 (1926).
[43] P. Lenard and S. Saeland, *Ann. d. Phys.*, **28**, 476 (1909).
[44] C. Ramsauer and P. Hausser, *Ann. d. Phys.*, **34**, 445 (1911).
[45] H. Scholl, *Ann. d. Phys.*, **14**, 193, 417 (1905).
[46] A. P. H. Trivelli, *Rec. trav. chim.*, **3**, 714 (1923).
[47] R. B. Wilsey, JOUR. FRANK. INST., **200**, 739 (1925).
[48] Graetz, *Ann. d. Phys.*, **29**, 314 (1886).

ver, etc.) are introduced. Röntgen [49] showed that a substance like sodium chloride, which in the pure state does not show any evidence of photo-conductivity, after exposure to X-rays (causing blue coloring), or when foreign substances in a highly dispersed condition were introduced into it, becomes photo-conductive. We know in photography the importance of the method of introduction of foreign substances in the silver halides. In fact, in general, the photo-conductivity seems to be sensitive to the method of introducing foreign substances.

A series of experiments is being planned in this laboratory on the photo-conductivity of silver halides and silver sulphide in connection with the photographic properties of both the fused substances and of single crystals. It is of interest to notice that along with the photographic intermittency effect and the failure of the reciprocity law, investigations have to be taken up to find a correlation with the photo-conductivity. If this correlation exists, it can be regarded as strong evidence for the existence of the now still hypothetical elementary photo-electric currents, which are supposed to circulate in the silver halide crystal during exposure. The other requirement is therefore to investigate the potential differences in silver bromide and in silver iodobromide between the foreign substances and the silver halide crystals during illumination, that means in general the distribution of the potentials.

SUMMARY.

(1) The fact that selenium and several other substances, including the silver halides and silver sulphide, are photo-conductive as well as photo-electric, taken together with the facts that in the silver halide grain silver sulphide and perhaps silver exists and that these appear to be more photo-electric than the surrounding silver halide, have led to the conclusion that the photo-electric effect and the photo-conductivity may play a part in the formation of the latent image.

(2) It is assumed that when light falls on the silver halide grain containing sensitizing specks of silver sulphide or silver, the greater photo-electric effect of silver sulphide or silver compared with that of the silver halide produces a difference in

[49] W. C. Röntgen, *Ann. d. Phys.*, **64**, 1 (1921).

potential between these and that in some way yet unknown the photo-conductivity produced by the light action causes an electrolytic deposition of silver in the neighborhood of the speck, hence increases its size and in consequence the developability of the grain. Such an effect does not preclude the simple photochemical decomposition of silver halide by light to give silver nor the possibility of silver sulphile serving as a halogen absorber.

(3) The mechanism of this photo-electric and photo-conductive effect is yet uncertain and requires considerable experimental work to determine its nature, but qualitatively there seems to be a correlation between the intermittency effect and also the failure of the reciprocity law and the latent image formation.

NOTES FROM THE U. S. BUREAU OF STANDARDS.*

INTERNATIONAL UNION OF SCIENTIFIC RADIO TELEGRAPHY.

DURING the present month the International Radio Conference is in session in Washington. In connection with this conference, the International Union of Scientific Radio Telegraphy will also hold its regular international meeting, these meetings taking place once in three years. The several international unions are organized under the International Research Council, and the American sections are under the National Research Council with headquarters in Washington.

The aims of the international union are: (1) To promote the scientific study of radio communication; (2) to aid and organize research work requiring international coöperation and to encourage the discussion and publication of the results; (3) to facilitate agreement upon common methods of measurement and the standardization of measuring instruments.

The international union itself is an organization framework for carrying on the international phases of the administrative work. The actual technical work is done by the various national sections, under the following officers: President—General G. Ferrie, France. Vice-presidents—Dr. L. W. Austin, United States; Dr. W. H. Eccles, England; Dr. Vanni, Italy; Dr. V. Bjerknes, Norway. General Secretary, Dr. R. B. Goldschmidt, Belgium.

In order to correlate the work of the several national sections, the international union has four international commissions on: (1) Methods of measurement and standards; (2) radio wave transmission phenomena; (3) atmospheric disturbances; (4) liaisons. The American section of the international union was organized in 1920, and is made up of an executive committee and five technical committees. The membership of the executive committee is as follows: Dr. L. W. Austin, chairman; representing the international union as its vice-chairman. Dr. J. H. Dellinger, technical secretary; representing the Department of

* Communicated by the Director.

Commerce. Dr. W. E. Tisdale, corresponding secretary; representing the division of physical sciences. Prof. J. S. Ames, representing the division of physical sciences. Major-General C. M. Saltzman of the Army. Dr. A. H. Taylor, representing the Navy. Dr. A. N. Goldsmith of the Institute of Radio Engineers. Members-at-large—E. F. W. Alexanderson, Major-General Geo. O. Squier, Dr. A. E. Kennelly, Dr. W. Wilson, Prof. E. M. Terry, F. Conrad.

The American section holds an annual meeting, at which progress reports are presented and technical papers read by various members of the committees.

The technical committees are as follows:

(1) Methods of measurement and standards.
(2) Radio wave transmission phenomena.
(3) Variations in radio wave direction.
(4) Wave phenomena above three megacycles.
(5) Atmospheric disturbances.

Two of these committees are headed by members of the staff of the Bureau of Standards.

The chairman of the first committee is Dr. J. H. Dellinger, chief of the radio section of the bureau. This committee is concerned with radio measurements and standards, particularly as regards measurement work performed by other committees of this organization. The committee promotes and participates in the improvement and standardization of methods for measuring frequency, field intensity of received waves, intensity of atmospheric disturbances, automatic recording devices, fading, and other types of radio measurements.

The committee on radio wave transmission phenomena is headed by Dr. L. W. Austin, in charge of the bureau's laboratory for special radio transmission research. This committee encourages and conducts measurements upon received field intensities. It investigates in connection with these measurements, diurnal and seasonal variations and effects of transmission under day and night conditions, sunset and seasonal effects, observation phenomena, etc. Special signals from high-power stations are employed in this work.

The chairman of the committee on radio wave direction is Dr. Gregory Breit, Department of Terrestrial Magnetism, Car-

negie Institution. This committee encourages and correlates research on the variations of wave direction with time, effects of topography on direction finder work, and the baring of polarization and other variations upon apparent observed phenomena.

The committee on wave phenomena above three megacycles is under the direction of Dr. A. H. Taylor, of the Naval Research Laboratory, Bellevue, D. C. The work covers such phenomena as daily and seasonal variations of received field intensities from high-frequency stations, character of fading and atmospherics, determination of skip distances, comparison of radio phenomena with magnetic and solar variations, weather, etc.

The committee on atmospheric distances, under Mr. H. T. Friis, of the Bell Telephone Laboratories, as chairman, promotes and coördinates measurements upon the intensity and direction of atmospherics, including daily and seasonal variations, characteristics of the various types of atmospherics, their frequency distribution, and methods of measurement, including recording devices.

Many radio problems can only be studied on a world-wide scale, so the international union plays a very important part in our progress toward a better understanding of radio and other allied natural phenomena. The great variety of work covered is illustrated by the titles of papers presented at the open session on October 13, as follows:

Employment in combination of photo-electric cells and lamps having multiple electrodes in the solution of various questions relative to problems of time measurement.

International comparison of frequency standards.

Precision determinations of frequency.

The Navy's primary frequency standard.

A radio-frequency oscillator for receiver investigations.

An automatic recorder for radio signals and atmospheric disturbances.

Experience in radio compass calibration.

Apparent night variations in crossed-coil radio beacons.

Ionization in the outer atmosphere of the earth.

The constitution of the earth's atmosphere up to great heights.

Ultra-short waves.

Height of the reflecting layer in August, 1927, and the effect of the disturbances of August 19.

The relation between radio reception, sun-spot position and area.

Solar activity and radio transmission.

TRANSMISSION OF VITALIZING RAYS BY GLASS AND GLASS SUBSTITUTES.

As the result of work in the bureau's radiometry section during the last few months, it has been demonstrated beyond all reasonable doubt that all the new glasses now being marketed for transmitting the ultra-violet "vitalizing" or "activating" solar rays, at 290 to 310 millimicrons, undergo a photochemical action, and decrease greatly in transparency to these activating rays when exposed to ultra-violet radiation.

The decrease in transmission of these glasses at 280 to 310 millimicrons is of the same nature whether the glass is exposed to the sun, the quartz-mercury lamp, or the carbon arc, the change in transmission being relatively more rapid for the mercury arc than for the sun.

Glasses which transmit 15 to 25 per cent. at 280 millimicrons when new decrease in transmission to less than 5 per cent. (depending upon the thickness and the amount of ferrous iron present as an impurity) after a few hours' exposure to the mercury arc or a few weeks to the sun.

After this drop to about 2 or 3 per cent. at 280 millimicrons, 15 to 20 per cent. at 303 millimicrons, and 30 to 45 per cent. at 313 millimicrons, prolonged exposure to the sun and to the mercury arc does not cause much further decrease in spectral transmission, as is to be expected since the photochemical action is fairly complete.

After this solarization has taken place these glasses probably still retain sufficient transparency to these vitalizing rays for therapeutic purposes. However, in all these glasses the transmission approaches rather close to the minimum limit that seems permissible in order to insure active stimulation of calcium metabolism, to prevent rickets, etc. The determination of this minimum limit in transmission for therapeutic purposes is a problem that will require further investigation by biologists.

ULTRA-VIOLET TRANSMISSION OF FABRICS.

During the past month further measurements have been made on close-weave and open-weave fabrics submitted to the bureau as approximately pure cellulose (viscose) and cellulose acetate rayons. After eliminating the light transmitted through the openings between the threads, the following ultra-violet transmission coefficients have been deduced for the (white, bleached, uncolored) threads:

Viscose varies from 16 to 27 per cent.
Cotton varies from 17 to 20 per cent.
Cellulose acetate varies from 11 to 29 per cent.
Silk varies from 14 to 18 per cent.
Wool varies from 5 to 15 per cent.

A slight coloring of the fabric by dyes, or yellowing with age, greatly decreases the transmission of the ultra-violet rays.

The bureau's tests on thin homogeneous colorless films of viscose rayon and on cellulose acetate rayon show that the latter is more opaque to the short wave-length ultra-violet rays, which is in agreement with the tests on the threads. The average viscose thread is more transparent than the average thread made from cellulose acetate.

After deducting for the openings between the threads, the transmission through the thread, especially when dyed, is only of the order of about 5 to 10 per cent. When one considers that the thread occupies from 95 to 99 per cent. of the total space, a transmission of only 10 per cent. of the total incident light is insignificant. Hence, it is apparent that in order to obtain beneficial therapeutic results, an open-weave fabric should be worn. Admitting this fact, it is apparent that it makes but little difference whether the thread is of cellulose acetate, cotton, wool, or silk. It seems evident that the importance of the composition of the material has been overestimated.

FINE ANNEALING OF OPTICAL GLASS.

It has been shown by investigators both in the United States and abroad that the index of refraction (and other physical properties of glass) are affected by the time and temperature at which the glass is annealed.

In preparing some glass for precision optical instruments, it

was found that the glass which was presumably well annealed showed variations from the average index of the magnitude of 37×10^{-6}. This variation was somewhat greater than was desired and the glass was re-annealed at a temperature of 486° C. for fourteen days. As a result of this treatment the variation from the average was reduced to 12×10^{-6}, or about one-third the original variation.

The heat treating or annealing was done in a specially constructed electrically heated furnace which consists essentially of a cylindrical cast-steel box thirty-two inches in diameter and six inches deep with heating elements above and below the box, the whole being insulated with eight inches of diatomaceous earth on all sides. The glass was placed in this furnace between two smaller iron plates in the box to insure greater uniformity of temperature distribution. Heat absorption measurements were made on a piece of the glass before re-annealing in order to determine the temperature at which the glass had been annealed previously so that the subsequent annealing would not materially change the index of refraction from the original value.

Analysis of the data obtained indicates that there were sufficient temperature gradients in the furnace in which the glass was originally annealed to produce the large variation found, and that during re-annealing there were still sufficient gradients, although they were much less than in the first case, to leave some optical inhomogeneity in the blanks. Results obtained in this experiment indicate that the greatest care must be taken to insure uniform temperature distribution if several pieces of glass are to be annealed and a uniform index of refraction obtained, and that if glass from the same melt is annealed at different times, it is necessary to duplicate annealing temperatures within a very few degrees.

PHYSICAL PROPERTIES OF SEMI-VITREOUS BODIES.

APPROXIMATELY four years ago the United States Potters Association began an investigation of the physical properties of semi-vitreous white-ware for domestic table use. The work was done at, and with the coöperation of, the Bureau of Standards. The report published as a limited edition by the potters association, is largely non-technical in character. It contains a résumé of the data obtained by physical tests of representative commercial

bodies, bisque fired and glazed in various ways, and also a discussion of the petrographic study of several of the bodies. This discussion is illustrated by excellent reproductions of photomicrographs. The following are some of the more important conclusions:

(1) American earthenware bodies differ more than is generally realized as regards their maturing temperature (which corresponds to firing to an absorption of about 8 per cent.). Some mature fully at cone 7, others at cone 9. (2) There are two periods of firing during which the absorption of the body is materially decreased, one between cones 4 and 5 (because of the presence of English ball clay), and one between cones 6 and 8 (because of feldspar). (3) Desired maturity of a body can be obtained by firing at a lower temperature (cone 6, for example), and then holding at this temperature until cone 8 is down. Soaking at cone 4 lowers the absorption at an average rate of 0.97 per cent. per hour, soaking at cone 6 lowers the absorption about 1 per cent. per hour, and soaking at cone 8 lowers the absorption about 1.47 per cent. per hour. (4) Glazed bodies usually are stronger mechanically than they are before glazing, but this presupposes a good "glaze fit." (5) In well-fired semi-vitreous bodies the fusion of the spar is complete, there is some surface attack of the spar on the flint particles and the local formation of fine crystals of mullite. (6) A well-fired glaze will show considerable attack upon the surface of the body, resulting in the growth of mullite needles into the glaze at right angles to the body surface.

SHEARING TESTS ON SAND-LIME BRICK.

In connection with an investigation of the strength of bond between mortar and sand-lime brick, a considerable number of physical tests have been made on the bricks used. It was believed that the results of the shearing tests would be of interest, inasmuch as there is only a limited amount of data of this nature to be found in the literature.

The apparatus used was designed at the bureau. In carrying out the test a cylindrical core is sheared out of the brick, the shearing strength being found by dividing the applied load by the area of the cylinder, exclusive of the ends. The sand-lime bricks had fairly smooth surfaces, so that a half brick could be

placed in the apparatus, and only an occasional one required a little grinding against another brick to make the surfaces fit properly.

There were 701 "normal" half bricks tested in this way, the "normal" half bricks being about 2¼ inches in thickness. The average shearing strength value obtained was 1055 lbs./in.^{2}, which was slightly less than one-half of the average value obtained in the compressive strength tests when the bricks were tested on edge.

In order to compare the effect of thickness of brick upon the shearing strength a further test was made on twenty-five brick, which gave an average shearing strength of 805 pounds for the "normal" half brick. The other halves were cut lengthwise to give sections about one inch in thickness, and these gave an average of 1650 lbs./in.^{2}.

CHARACTERISTICS OF SEVERAL HEAVY CLAYS.

THE Columbus branch of the bureau is conducting an investigation to determine the properties characteristic of various clays and shales used in the heavy clay industries. The object of most of the experimental work to date has been to determine the various physical properties of the unfired clays and shales, representative samples of which were obtained from various localities in Ohio. As indicated in *Technical News Bulletin No. 119* (March, 1927), the various test pieces were cut from a regular brick size column extruded from a small auger and pug mill. The various clays and shales were ground in a regular commercial five-foot dry pan to pass a 12-mesh screen.

Although most of the shales show no great variation in grain size and sandiness, it is interesting to note that the slaking time varies from about four to thirty-five minutes. As would be expected, the sandiness and grain size of the alluvial clays vary considerably, and the slaking time runs from about ten minutes to several hours. Although only about 50 per cent. of the shales pass a 260-mesh screen, against about 95 per cent., for the alluvial clays, there appears to be present in the shales considerable material of extremely fine or colloidal dimensions, since the water soon becomes very murky, while remaining clear in the case of the alluvial. The screen analysis apparently indicates the general

workability and texture which are to be expected in running the clay or shale through an auger machine. The following table shows a typical wet-screen analysis on an alluvial clay and a shale:

Sieve.	Alluvial Clay. Per cent.	Shale. Per cent.
Retained on 60	2.7	38.0
" " 100	3.5	4.2
" " 150	2.4	1.4
" " 200	1.2	0.7
" " 260	2.9	1.1
Through 260	87.4	54.6

Although the tests have not been completed, Table I shows some of the average values obtained for several Ohio shales and alluvial clays. As one would expect, the alluvial clays required considerably more water to temper them properly than did the shales. Because of the incompleteness of the data, no correlation has been attempted.

Factor.	Green Shales. Min.	Green Shales. Max.	Green Alluvial Clays. Min.	Green Alluvial Clays. Max.
Slaking time, minutes	4.4	33.0	10.0	150.0
Volume shrinkage, per cent.	10.8	21.6	18.1	26.0
Calculated linear shrinkage, per cent.	3.6	7.6	6.3	9.4
Tempering water, per cent.	19.3	23.7	24.2	31.3
Modulus of rupture, lbs./in.^{2}	131.7	316.3	141.7	518.2
Porosity, per cent.	23.3	31.4	26.2	35.1
Apparent specific gravity..	2.58	2.75	2.61	2.76
Impact strength, gm./cm....	2.14×10^5	4.32×10^5	1.60×10^5	5.76×10^5

BOND BETWEEN CONCRETE AND HOLLOW TILE.

AN investigation of the bond between concrete and hollow tile has just been completed. The factors covered by the tests included six different types of hollow tile, and concrete mixtures of several consistencies and proportions. The tiles used in making the specimens were in a dry, saturated, or in a semi-saturated condition. The specimens were cured either in dry or in damp storage. All specimens were tested when twenty-eight days old, the damp storage specimens being allowed to dry out fourteen days prior to testing. Concrete control cylinders 6×12

inches were made from the same batches as the concrete in each bond specimen.

The test specimens represented sections from a hollow tile concrete floor, each made up of two tiles joined by a concrete block 4 inches in thickness. The testing consisted in loading the concrete blocks by a heavy bearing block, the tiles forming the lower base. The desire was to obtain a shearing failure between the concrete. The results obtained indicate that the bond depends largely on the strength of the concrete, but this relation may be disturbed by using saturated tiles or damp curing. The condition least favorable to a good bond was found to exist when specimens were made from saturated tiles and then cured damp. This particular tile had an absorption of about 10 per cent. In general, damp curing did not increase the bond strength.

Specific factors which affected the bond strength were:

(1) Strength of the concrete, the stronger the concrete the greater the bond.

(2) Absorption of the tile, no general law being observable.

(3) Amount of water in the tile, the greatest bond being developed by dry tile, slightly less by sprinkled tile, and the least bond by saturated tile. For tiles of the lowest absorption about 3.1 per cent., there was no material strength difference in bond, whereas the greatest bond difference was recorded for the tiles with 5 + per cent. absorption. The absorption ranged up to 21 + per cent.

(4) Curing conditions. The dry-cured specimens developed a slightly higher average strength than the damp-cured.

As a practical guide in construction, based on the results of these tests, it is recommended that the hollow tiles be sprinkled only enough to work off the dust and loose particles, and that the concrete contain the minimum amount of mixing water necessary for its proper placement.

A paper is being prepared for publication which will give the results of these tests in greater detail.

CITY PLANNING AND ZONING.

THE division of building and housing recently issued in mimeographed form a report on zoning progress in the United States which is up to date as of July 1, 1927, superseding a simi-

lar report issued last April. The number of zoned municipalities recorded in the 1926 report was 436, representing a population slightly in excess of 27,500,000, while the number recorded in the 1927 report is 553, representing a population of over 30,000,000.

The highest courts in twenty-two states have now upheld zoning ordinances. Permission of municipalities to zone is embodied in the laws of forty-six states, according to a recent list prepared by the division, and twenty-eight state laws now include all or a large part of the Standard State Zoning Enabling Act, which the department issued in 1924.

Lists of zoned municipalities arranged according to population have been brought up to date and are available for distribution. Requests should be addressed to the division of building and housing, Department of Commerce.

In this connection it is interesting to note that the City of Cincinnati has given the city planning commission jurisdiction over the acceptance and layout of all new streets, subject only to a different decision by a two-thirds vote of the city council, as proposed in the Standard City Planning Enabling Act.

DRUGGISTS' RESEARCH BUREAU.

There has recently been formed a Druggists' Research Bureau, with offices in New York City, to act as a national clearing house for obtaining facts necessary to the welfare and success of the drug industry.

The committee in charge consists of A. K. Mayer, of Indianapolis, chairman; J. F. Finneran, of Boston, vice-chairman; G. B. Evans, Albany, N. Y.; Sidney Hollander, Baltimore; Ambrose Hunsberger, Philadelphia; H. S. Noel, Indianapolis; P. C. Olsen, Philadelphia; Alf. W. Pauley, St. Louis; J. T. Woodside, Chicago; and E. L. Newcomb, secretary.

The bureau gathers facts making for the better conduct of the industry. These include data on simplification of stock to increase turnover and increase profits; simplification and standardization of cost accounting; and simplification and standardization of arrangement of stock to expedite service within the drug store and eliminate waste. In the research work along the lines of simplification, the division of simplified practice will coöperate with the Druggists' Research Bureau.

The Dependence of the Dielectric Constants of Gases and Water Vapor on Pressure When This Pressure Is Small. KARL WOLF. (*Ann. der Physik*, No. 14, 1927.)—The connection between the pressure of gases and their dielectric constants has been but little examined in the pressure range below one atmosphere. Further, Glaser found just in this pressure interval a departure from a linear relation between gas pressure and magnetic susceptibility in the case of certain diamagnetic gases. It seemed rather likely that a similar divergence should exist for the dielectric constant, provided Glaser's results are reliable.

Two electric circuits carrying high-frequency currents were so coupled as to cause in a telephone audible beats. One of the circuits included a condenser containing the gas to be examined, while the second circuit was unchanged. The change in the dielectric constant of the variable gas was proportional to the change in the number of beats per second. For hydrogen, nitrogen, oxygen, carbon dioxide and ammonia gases the pressures were plotted as abscissas against ordinates proportional to the excess of the measured dielectric constant over that of vacuum, *i.e.*, unity. In all these the curves were strictly straight lines, showing a linear connection between gas pressure and dielectric constants. When water vapor was investigated the corresponding curve was found to consist of two connected straight lines, and it required a lengthy examination to prove that this change of direction in the line was due to conductivity in the insulating material of the condenser. Even with such excellent insulators as amber, quartz and glass the resistance was discovered to decrease rapidly when the pressure of unsaturated water vapor was increased. Of the five gases maintaining the linear relation the dielectric constant of ammonia increased most rapidly and that of hydrogen least rapidly with increase of gas pressure. Nitrogen and oxygen changed at about the same rate, the former being slightly ahead of oxygen. G. F. S.

Chlorides and Tobacco.—The influence of chlorides in the fertilizer upon the burning quality of tobacco is discussed by E. H. JENKINS (*Conn. Agric. Exp. Sta. Bull.*, **282**, 92–95, 1926). New England tobacco is used for wrapper and binder in the form of leaf, and must have a good "burn." This leaf "must hold fire well, not burning too fast or too slowly. It must not cool on the cigar and it must leave a clear white or light gray—not 'muddy'—ash." While the tobacco requires a small quantity of chloride (muriate) for its normal development, yet potassium chloride, used as a fertilizer, imparts bad burning quality to the crop. Some salt of potassium other than the chloride, *e.g.*, the sulphate or the nitrate, should be used as the source of potassium in the fertilizer. However, the small quantities of chlorides present as impurities in the other commercial salts of potassium are not likely to be injurious to the tobacco. J. S. H.

NOTES FROM THE RESEARCH LABORATORY, EASTMAN KODAK COMPANY.*

PHOTOGRAPHIC SPECTROPHOTOMETRY IN THE ULTRA-VIOLET REGION.[1]

By Loyd A. Jones.

THE characteristics of photographic materials which must be considered when used for the precise measurement of radiation intensities are discussed. It is pointed out that such factors as the intermittency effect, failure of the reciprocity law, variation of gamma with wave-length, non-uniformity of effective sensitivity, and non-uniformity of development may cause serious errors unless the method adopted and the technique followed are specifically designed to eliminate these errors. The criterion of equality between two radiation intensities is that equal densities shall be produced by equal exposure times, it being assumed that the radiations in question are of the same wave-length or of identical wave-length composition, and that exposure is non-intermittent. A method is described which meets the theoretical requirements. The instrument required for the application of this method is also described. The application of photographic spectrophotometry to the measurement of solar radiation in the ultra-violet region is discussed and some suggestions made for the development of a method free from objectionable features.

THE MECHANISM OF THE WURTZ-FITTIG REACTION.[2]

By W. E. Bachmann and H. A. Clarke.

THE action of sodium upon boiling chlorobenzene leads to the formation of 20–25 per cent. of the theoretical yield of biphenyl, benzene and hydrocarbons of high molecular weight being formed simultaneously. The following hydrocarbons have been identified among the reaction products: *o*-diphenylbenzene,

* Communicated by the Director.

[1] Communication No. 314 from the Kodak Research Laboratories, and published in *Bull. Nat. Res. Council*, No. 61, Washington, D. C., 1927.

[2] Communication No. 302 from the Kodak Research Laboratories and published in *J. Amer. Chem. Soc.*, 49, 2089, 1927.

p-diphenylbenzene (trace), triphenylene, *o.o'*-diphenyl biphenyl. The identity of the *o*-diphenylbenzene and of the *p*- and *o'*-diphenyl biphenyl has been confirmed by synthesis. Analogous results have been obtained with alkyl halides. Thus, *n*-heptyl bromide on treatment with sodium yields, besides tetradecane, heptane, heptylene, and a heneicosane.

When the reaction was carried out in presence of toluene, the proportion of benzene formed was increased, and the toluene was found to have entered into the reaction. A similar result was obtained with dimethylaniline. These results are interpreted by assuming the intermediate formation of free radicals.

NOTES FROM THE U. S. BUREAU OF MINES.*

FLOTATION OF ORES.

GREAT values are constantly being saved by the flotation treatment of certain ores. However, present practices are largely the result of cut-and-try methods rather than experimentation on the fundamentals of the operation. Realizing the need for fundamental data on which to base improvements in practice, the Bureau of Mines, through its Intermountain Station at Salt Lake City, is conducting research on four phases of such flotation fundamentals. They include (1) the effect of grain size on differential flotation; (2) effect of finely disseminated solids (slime) on the differential separation of lead, zinc, and iron sulphides; (3) flotation of oxidized copper minerals, and (4) flotation of sulphides of copper and their separation from the sulphides of iron. The great difficulty in attacking the first problem is to secure in quantity accurately sized material below 200 mesh. Results so far obtained show that the fine grains of minerals that are substantially equally floatable are floated ahead of the coarse grains under suitably controlled operations. In the second problem the flotative properties of sphalerite are being studied. After the oxidized surfaces have been cleaned the pure sphalerite is readily floatable. The flotation of malachite with various saturated fatty acids and the study of depressing agents was begun under the third problem. The fourth problem has been attacked by a study of fundamental flotative properties of calcite, particular attention being paid to collecting agents.

THE DETECTION OF SULPHUR IN PETROLEUM AND PETROLEUM DISTILLATES.

By F. W. Lane and John M. Devine.

SOME months ago the Bureau of Mines began research of a fundamental character on sulphur in petroleum. In the course of this work, the usual qualitative test for sulphur in organic compounds was applied to certain oils that were believed to contain approximately 0.5 per cent. sulphur. No sulphur was found, and, as a check on the qualitative test, the oils were subjected to quantitative determination of this element by the oxygen-bomb

* Communicated by the Director.

method. The sulphur content of the oils ranged from 0.10 per cent. to 0.50 per cent. This experience led to further trial of the qualitative test for organic sulphur, both on the oils mentioned above, and on common sulphur-containing organic substances. No difficulty was experienced in detecting the element in the latter, but with the petroleum oils the test, as usually performed, could not be relied upon to detect less than 0.60 per cent. of sulphur consistently.

The use of mechanically polished pure sheet copper for the detection of free sulphur, hydrogen sulphide, and perhaps other sulphur compounds has been recommended.

As it is recognized that this test cannot be relied upon to show the presence of all sulphur compounds in oils, attention was concentrated on the usual qualitative test for sulphur as applied to organic compounds.

Therefore, experiments were conducted and a test for the qualitative detection of sulphur was developed that differs somewhat in detail from the usual procedure. Each step in the procedure has been carefully studied. Assuming that the petroleum medicinal oil used in the above tests contained about 0.01 per cent. sulphur, it is probably fair to state that the test will detect sulphur in the elementary or combined form in petroleum oils when present to the extent of only 0.02 to 0.03 per cent. This is much more sensitive than the test as usually carried out, using sodium nitroprusside. Further details are given in Serial 2828.

PRODUCTION OF HIGH-ALUMINA SLAGS IN THE BLAST FURNACE.

By T. L. Joseph, S. P. Kinney, and C. E. Wood.

THE Bureau of Mines has been conducting a series of investigations relating to the smelting of iron ores for the past ten years. One of the early investigations was prompted by the apparent need for more information regarding the physical properties of blast-furnace slags. As a result of systematic measurements, the viscosity of blast-furnace slags was determined over a wide range of composition and temperature. One of the points covered in the slag viscosity studies was the effect which varying percentages of alumina have upon viscosity. Further details are given in a forthcoming report of the Bureau of Mines.

THE FRANKLIN INSTITUTE.

(Proceedings of the Stated Meeting Held Wednesday, October 19, 1927.)

THE regular monthly meeting of the Institute was called to order at eight-fifteen P.M., by Mr. Walton Forstall, Vice-president of the Institute, in the absence of the President, Dr. Wm. C. L. Eglin, who was prevented by illness from attending.

The Chairman called for the reading of the minutes of the last meeting. The Secretary stated that the last meeting had been the Medal Day meeting, held in May, and that the minutes of that meeting had been printed in the June number of the JOURNAL of the Institute. He therefore moved that the minutes be approved as printed. This motion was unanimously adopted.

The Chairman called for the reports from several committees. The Secretary reported from the Committee on Meetings that an effort would be made to have at least one lecture each month which should display by means of experiments and demonstrations some new development in science or engineering. This practice was being inaugurated at this meeting by a lecture on the gyroscope. The other lectures during the month would be of a more special and technical character, but any or all of them might be accompanied by experiments and demonstrations. The list of meetings up to January first has been sent to all members of the Institute. The lecture program for the balance of the year is now in course of preparation.

The Secretary made a statement for the information of the members concerning the activities of the Bartol Research Foundation. Announcement was made at the last meeting of the Institute of the election of Dr. W. F. G. Swann, former head of the Department of Physics at Yale University, as Director of the Bartol Research Laboratory. Since that time Doctor Swann has taken up active work at the laboratories, where fundamental researches are being vigorously pursued by eight fellows of high ability and with promising records. The outlook at present for successful research work is much more favorable than it has ever been before.

The Secretary stated further that an arrangement had been entered into with Swarthmore College for the leasing to the Institute of a tract of land on the college campus, upon which the Bartol Research Laboratory is to be erected. Plans for this laboratory are now being drawn, so that it is hoped that ground will be broken early in 1928 and that the building will be completed in time for research work in the autumn of next year. The plans in preparation call for a thoroughly fire-proof laboratory of handsome design and generous size, in which all of the activities of the Foundation will be carried on. The Institute is to be congratulated heartily upon the favorable outlook for successful work at the Foundation.

For the Membership Committee, the following report was submitted: There were twenty-four additional members—one life member, twelve resident

members, ten non-resident members, one student member; twelve members have resigned and fifteen have been lost through death.

The Secretary announced finally that Prof. Wilder D. Bancroft, head of the Department of Chemistry of Cornell University, has accepted an invitation to deliver the Christmas Week lectures on the James Mapes Dodge Foundation, and that Professor Bancroft's subject will be "Combustion, Comfort and Color." Full details concerning these lectures will be announced later.

The Chairman introduced as the speaker of the evening Mr. Elmer A. Sperry, President of the Sperry Gyroscope Company, inventor of the gyroscopic compass, and one of America's great inventors. Mr. Sperry showed in a most interesting and entertaining fashion the development of the gyroscope and some of its uses, and illustrated by means of demonstrations with models the working of the various gyroscopic instruments.

The lecture was followed by some discussion and questions, and an examination close at hand of the models which had been shown.

The meeting adjourned at nine-forty-eight P.M., with a rising vote of thanks and of hearty appreciation for Mr. Sperry's splendid talk.

HOWARD MCCLENAHAN,
Secretary.

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting Held Wednesday, October 5, 1927.)

HALL OF THE COMMITTEE,
PHILADELPHIA, October 5, 1927.

MR. M. M. PRICE *in the Chair.*

The following reports were presented for first reading:

No. 2867: Kinematic Computing Devices.

No. 2873: Method for Making and Finishing Welded Pipe.

GEO. A. HOADLEY,
Secretary to Committee.

MEMBERSHIP NOTES.

ELECTIONS TO MEMBERSHIP.

(Stated Meeting, Board of Managers, October 19, 1927.)

RESIDENT.

MR. PAXON DEETER, Lawyer, 1718 Packard Building. For mailing: 18 South Bryn Mawr Avenue, Bryn Mawr, Penna.

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DR. NICOL HAMILTON SMITH, Research Chemist, Solidifier Corporation. For mailing: 3036 North Seventh Street, Philadelphia, Penna.

MR. B. W. STORY, Chemist, Vacuum Oil Company, Research Department. For mailing: Vacuum Oil Company, Paulsboro, N. J.

NON-RESIDENT.

MR. BERNARD ARTHUR BEHREND, Consulting Engineer, 110 Cliff Road, Wellesley Hills, Mass.

MR. WILLIAM H. BRISTOL, President, The Bristol Company, Waterbury, Conn.

MR. JOSEPH D. R. FREED, Radio Engineer, President, Freed-Eisemann Radio Corporation, Junius Street and Liberty Avenue, Brooklyn, N. Y. For mailing: 912 Fifth Avenue, New York City, N. Y.

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The Leeds and Northrup Company:

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CHANGES OF ADDRESS.

COLONEL GEORGE L. ANDERSON, Army and Navy Club, Washington, D. C.

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MR. GEORGE W. BORTON, Liberty Trust Building, Broad and Arch Streets, Philadelphia, Penna.

MR. BENJAMIN J. DASHIELL, 405 Central Avenue, Towson, Md.

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MISS EMILY E. HOWSON, 415 Sterling Place, Madison, Wis.

MR. J. P. JEFFERSON, 21—The Fairway, Santa Barbara, Calif.

DR. JOHN JOHNSTON, U. S. Steel Corporation, 71 Broadway, New York City, N. Y.

MR. JONATHAN JONES, P. O. Box 1594, Pittsburgh, Penna.

MR. JOSEPH B. KING, JR., Alden Park Manor, Wissahickon and Chelten Avenues, Germantown, Philadelphia, Penna.

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MR. ROBERT L. WOOD, 30 East Bells Mill Road, Chestnut Hill, Philadelphia, Penna.
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NECROLOGY.

Prof. Svante August Arrhenius, Nobel Institute, Stockholm, Sweden.
Mr. Horace A. Beale, Jr., Parksburg, Penna.
Mr. A. Van Wyck Budd, Berwyn, Md.
Mr. Joseph M. Gazzam, Philadelphia, Penna.
Mr. Victor Lenher, Madison, Wis.
Mr. George L. Norris, Wilmington, Dela.
Prof. H. P. Talbot, Cambridge, Mass.

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WHIPPLE, HARVEY (compiler).—How To Do It: A Book of "Kinks" from the Magazine "Concrete." 1919.

BOOK REVIEWS.

CHEMICAL ENGINEERING CATALOG. Twelfth annual edition, 1160 pages, illustrated, quarto. New York, The Chemical Catalog Company, Inc., 1927. Prices adjusted to special conditions as indicated in the volume.

This publication has been noticed as it appeared in the issues of the JOURNAL and it is necessary here only to record that the present edition maintains the character of the work as established in its earlier publications and constitutes a very valuable summary of the chemical engineering manufacturing in this country. The classified directory has been further enlarged and contains the listings of more than 2000 manufacturers. Its use to those engaged in purchasing the materials of chemical engineering is very great and the manner in which the publishers keep in touch with those who use the book is highly commendable. The information bureau maintained by the publishers for many years has been doing excellent service to its customers. More than 2400 books are listed in the technical and scientific books section. Information is also given concerning the American Chemical Society's Monographs. The book is printed in excellent form and very liberally illustrated.

HENRY LEFFMANN.

STANDARDS AND TESTS FOR REAGENT AND C.P. CHEMICALS. By Benjamin L. Murray. Second edition, revised and enlarged. xiii–560 pages, 8vo, cloth. New York, D. Van Nostrand Company, Inc., 1927. Price, $5.

One of the most important points in chemical analysis is to be assured that the reagents are exactly what they are intended to be. A very large amount of error has occurred in consequence of overlooked impurities. Sometimes such condition may result to the advancement of science. Such is the case in the test for arsenic suggested by Reinsch. He was experimenting on the action of hydrochloric acid on copper and obtained a very distinct deposit upon the metal. It appeared, however, that his hydrochloric acid was impure, containing a notable amount of arsenic and thus he was led to discover a reaction with this important element and with a number of others also of importance. Reinsch's test thus became to be one of our most important preliminary tests in toxicology. One wonders when this procedure would have been discovered if Reinsch had had a pure acid for his experiment. It is also generally known that the test which Kekulé devised for benzene is really due to the presence of a sulphur compound originally unsuspected.

There was much laxity in earlier years in the preparation of reagents and the term c.p. covered some sins, being often merely indicative of an approximate purity. The progress of applied chemistry, however, steadily increased the necessity of precision both as to the detection of substances and their determination. Within a comparatively recent period, therefore, there have grown up two well-developed systems of controlling chemical analysis: The establishment of standard methods and the manufacture of analyzed reagents. The work in hand is a second edition and the preparation of it has involved the revision and improvement of the standards and tests for reagents, which was the only feature of the first edition, and the introduction of considerable

new material for pure chemicals. The modifications of the methods of testing are not very numerous, but they have been obtained by inquiry among the users of the book. Standards of purity and methods for testing over 200 c.p. chemicals have been introduced. The book represents a very considerable extension of the material that was first presented by Krauch in 1870. As noted above, the author in the preface refers to a lack of agreement between consumers, dealers and manufacturers as to what the expression c.p. means and the author has aimed to secure and present information which may serve to eliminate these confusions. The standards set up are essentially practical, that is, they do not aim at absolute purity but to secure tests which will give to each worker the reagents that are adapted to the problems in hand. The descriptions are comprehensive and in full detail and the book therefore offers to the practical chemist a means of checking up all the materials which are used in analytic routine.

HENRY LEFFMANN.

MODERN ECLIPSE PROBLEMS, being the Halley Lecture delivered on 20 May, 1927, with notes for the observation of the eclipse of 29 June, 1927. By F. J. M. Stratton, D.S.O., M.A., Fellow of Gonville and Caius College, Cambridge. 34 pages, illustrations, 8vo, paper. New York, Oxford University Press, American Branch, 1927. Price, $.85.

Eclipses have excited the wonder and even the fear of mankind since the earliest period of recorded history. It is greatly to be regretted that the ancient philosophers did not make exact records of the time and places of eclipses in their relation to human events, for modern historians would have a most excellent method of checking up chronologic data by this aid. Here and there we have notes of important eclipses, but it appears from the number that must have occurred that but a scant record has been transmitted. As merely a side issue, it is worth noting that the appearances of comets have been occasionally recorded, and it is fortunate that a considerable number of returns of Halley's Comet are now known and accurately dated.

In honor of Halley a lecture is now delivered and the present pamphlet, as is stated, refers to the work done in connection with the eclipse of last May. It appears from the introduction that only once in the last thousand years has the belt of totality of a solar eclipse passed across Oxford (in 1715). We may contrast the imperfect methods of observation of eclipses for several thousand years up to recent times when the spectroscope and the camera with other special instruments enable far more information to be obtained. We have now indeed through the motion picture method the ability to reproduce the phenomena of the eclipse to every one and everywhere. The present pamphlet furnishes information on many points concerning this recent eclipse visible in England and is illustrated with photographs of various phases. One photograph shows the stars whose displacement verified the theory of relativity. This is an abstruse subject but is part of the data that have been accumulated. As the author remarks, the attitude of civilized mankind toward eclipses has ceased to have any element of superstition or fear and eclipses of the sun have become important as throwing light upon certain problems in physical astronomy.

HENRY LEFFMANN.

LABORATORY MANUAL OF GENERAL INORGANIC CHEMISTRY. By M. Cannon Sneed, Professor of Chemistry, University of Minnesota, and Raymond E. Kirk, Assistant Professor of Chemistry, University of Minnesota. xvii–181 pages, illustrations, 12mo, cloth. Boston, Ginn and Company, 1927. Price, $1.20.

This is a book of a type of which there are very many examples. It is written as many such books are, as a practical companion to a text-book of chemistry. It begins with an introduction on chemical manipulations, liberally illustrated with somewhat sketchy drawings, but goes into considerable detail in quantitative work. All such works require careful personal supervision in the laboratory. The book is limited to inorganic chemistry and contains a very considerable amount of information on that phase of laboratory work. Considerable attention is also given to the more modern theories of chemical action. Indeed, there is much more matter than is usually placed in the laboratory manual, the explanations of chemical principles being generally relegated to the text-book. Several tables of chemical data are appended, including a list of atomic weights which does not contain several of the most recently discovered elements. This omission, however, is of little significance in a work of this type. HENRY LEFFMANN.

LES IONS D'HYDROGÈNE, SIGNIFICATION, MESURE, APPLICATIONS, DONNÉES NUMÉRIQUES. Par W. Kopaczewski. ix–322 pages, illustrations, 8vo, paper. Paris, Gauthier-Villars et Cie., 1926. Price, 60.20 francs, carriage free.

Abstract from author's introduction and brief table of contents:

Insofar as applied to measure degree of acidity, the conception of hydrogen-ion concentration incontestably constitutes a real advance; it is logical and experimentally true. Unfortunately the evaluation of this concentration is not yet beyond all criticism. Based upon the electromotive force of "concentration cells," the method originates with the conceptions of Arrhenius and upon the ideas of Nernst upon the tension of solution, but these conceptions appear more and more to be very approximate expedients.

This treatise is intended to meet the needs of readers who seek practical processes of making rapid determinations with a degree of precision within the capacity of the method, and those whose aim is to acquire a thorough knowledge of the method, or again those who may have both objects in view. To meet these requirements, three styles of type are employed. Readers who desire only theoretic generalities will find *résumés* in heavy type while for detailed development, the type of the body of the work is to be followed. The critical or doubtful portions of the subject are in small type. Finally the bibliography enables the reader to consult original sources.

The aim has been to make the account of methods of measurement more complete than any previously available. Not only are electrometric and colorimetric methods discussed in full, but there are included old and entirely forgotten methods as well as the most recent, such as the use of triode-amplification, catalysis and the zero-method with the telephone. Major stress is laid upon the practical details of the electrical method, the basis of measurement

of actual acidity. The colorimetric method is carefully examined and its value discussed and defined. Doubtful indicators have been eliminated and over-simplified technique which is therefore unreliable has been entirely omitted.

In view of the very adequate treatment which biological and medical applications have received in other hands, these divisions of the subject have been only briefly considered. The bibliographical references which accompany each chapter have been utilized in developing the subject-matter therein and may be profitably consulted by those who desire further amplification. The numerical data which are included have been carefully compiled from the most recent works and carefully verified.

First Part, Theoretical considerations, Chapter I, Electric dissociation; Chapter II, Equilibrium in electrolytic solutions; Chapter III, Actual acidity. Second Part, Measure of concentration in hydrogen ions, Chapter I, Electrometric method; Chapter II, Colorimetric method; Chapter III, Special Methods and technique. Third Part, Applications and results, Chapter I, Chemical applications; Chapter II, Biological applications; Chapter III, Medical applications; Chapter IV, Numerical data. L. E. P.

CONDUCTIBILITÉ ÉLECTRIQUE DES MÉTAUX ET PROBLÈMES CONNEXES. Rapports et discussions du quatrième conseil de physique tenu à Bruxelles du 24 au 29 Avril 1924 sous les auspices de l'Institut International de Physique Solvay. viii–367 pages, 25 × 16.5 cm., paper. Paris, Gauthier-Villars et Cie., 1927. Price, 50 francs.

The Solvay International Institute of Physics, since its inception in 1912, has already been the medium of dissemination of a notable collection of papers dealing with fundamental problems of physical science which have been presented at its yearly meetings. The present volume contains the papers brought before the society at its fourth meeting in 1924. There are nine papers contributed by as many representative investigators with the discussions in full. These papers and the discussions are all reported in French, although some were presented in the language of their authors. The list is as follows:

H. A. Lorentz, Application of the theory of electrons to the properties of metals.

P. W. Bridgman, Report on the phenomena of conductibility in metals and their theoretic explanation.

O. W. Richardson, Another theory of metal conductibility.

W. Rosenhain, The internal structure of alloys.

Witold Broniewski, Electric resistance and expansion of metals.

A. Joffé, The electric conductibility of crystals.

Kamerlingh Onnes, New experiments with supra-conductors.

Edwin H. Hall, Metallic conduction and the transverse effects of the magnetic field.

A. Joffé and N. Dobronravoff, The propagation of the impulses of a radiation.

The analysis of the important subject of electric conductivity of metals by present-day physical theories in the hands of these leading investigators can scarcely fail to open the way to new advances. LUCIEN E. PICOLET.

A GUIDE TO THE LITERATURE OF CHEMISTRY. By E. J. Crane, Editor of *Chemical Abstracts*, and Austin M. Patterson, Professor of Chemistry, Antioch College. 438 pages, 8vo. New York, John Wiley and Sons, Inc., 1927. Price, $5.

The literature of chemistry is now enormous and is growing at a rapid rate. Chemistry itself has spread into so many phases and interlocked with so many other sciences that it is somewhat difficult to determine the limits to which a book like this should be confined. It is not a mere collection of titles or periodicals with data concerning their frequency of publication and scope, but an essay on the general nature of the literature intended to aid the research worker in his efforts to get data before proceeding upon any line of investigation. There is so much anticipation in earlier literature that it is very important that the research worker should adopt the motto which has been given for another class of literature and for a different purpose, namely to "search the scriptures." Many a time a consultation of journals or text-books many years old will show anticipation or at least such information as will render some investigation unnecessary. At the same time the earlier workers were liable to error. They did not have the apparatus that are in our hands to-day nor did they have the precision of methods that are now in vogue. In chemistry two developments have occurred within the past quarter century which have been of great advantage in determining the accuracy of both routine work and research. These are the introduction of standard methods of analysis and analyzed chemicals.

Turning to the book in hand, there is first a chapter on the problem and purposes of searches. Much information is given here as to the manner in which consultation of the literature should be made. Of course language differences present a very serious difficulty. As the author remarks, a reading knowledge of French and German is absolutely necessary to the English-speaking chemist who wishes to keep up with his science. Not sufficient attention is paid to this phase of education in our colleges, and many students are graduated with degrees of considerable importance who yet cannot read satisfactorily the journals and text-books of France and Germany. The scientific literature of these two countries is comparatively simple, the other literature, that is, in philosophy and economics as well as the dramatic, poetic and fictional productions, represent more difficult problems. Unfortunately, in spite of the efforts that have been made of recent years to secure a universal language, no satisfactory result has been obtained, and development of chemical activities in nations that up to a comparatively few years were unimportant has added to the linguistic troubles. Italian does not present a very great difficulty and its literature other than scientific is sufficiently attractive to make a study of the language worth-while; but we now have added the Scandinavian tongues, Dutch, Russian and Japanese as well as a certain amount of Spanish contributions.

Information is given concerning the leading text-books, also classifications of journals by subjects. Explanation is given upon nomenclature and about sixty pages are devoted to an alphabetical list of journals, after which follow lists of dealers and select lists of books classified by their subjects, the whole followed by a good index.

HENRY LEFFMANN.

ATOMIC THEORY, AN ELEMENTARY EXPOSITION. By Arthur Haas, Ph.D., Professor of Physics in the University of Vienna, translated by T. Verschoyle, M.C., Ph.D., A.R.C.S. xiv–222 pages, 22 × 14 cm., cloth. New York, D. Van Nostrand Company, Inc. Price, $3.50.

The task of the student of physics has never in modern times been a light one and in recent years the burden has increased to formidable proportions. Advances have been following each other in amazingly rapid succession until it seems that the magnitude of this accumulation of intellectual wealth must take it out of reach of all but those of super-intellectual capacity.

In his Introduction to Theoretical Physics, the English translation of which was noted in these pages last year, Professor Haas admiringly sets forth the elements of modern physical theory in which he discusses at length modern atomic theory about which a vast development is taking place. That division of the subject is included in Part III of the second volume of the above work. The present volume is a separate issue of that part of the work for readers who want a clear and readable exposition of the rapid progress in modern atomic theory. In the time which has elapsed between the appearance of the German and the present English editions, much new work has been accomplished in atomic theory, and with the coöperation of Professor Haas the editor has been able to bring the volume as up-to-date as can reasonably be expected.

The subject is presented under five capital divisions: Elementary Quanta, Theory of the Hydrogen Atom, Röntgen Rays, Theory of the Elements, General Theory of Spectra and of Atomic Structure. Under these captions the reader will find the whole development of modern advances in physical science, some of it in mathematical language but most of it in a descriptive form which is certain to enlist and hold the reader's interest and, if the phenomena discussed and their applications are new to him, arouse his admiration for achievements which may well furnish inspiration for a modernized version of the "wonders of science." There is no trace of foreign idiom in the translator's elegant rendition. LUCIEN E. PICOLET.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS. Report 269, Air Force Tests of Sperry Messenger Model with Six Sets of Wings, by James M. Shoemaker. 20 pages, illustrations, diagrams, quarto. Washington, Government Printing Office, 1927. Price, ten cents.

The purpose of this test was to compare six well-known airfoils, the R.A.F.15, U.S.A.5, U.S.A.27, U.S.A.35-B, Clark Y, and Göttingen 387, fitted to the Sperry Messenger model, at full-scale Reynolds Number as obtained in the variable density wind-tunnel of the Committee and to determine the scale effect on the model equipped with all the details of the actual airplane. The results show a large decrease in minimum drag coefficient upon increasing the Reynolds Number from about one-twentieth scale to full scale. Maximum lift coefficient was increased with increasing scale for all the airfoils except the Göttingen 387, for which it was slightly decreased. A comparison is made between the results of these tests and those obtained from tests in this tunnel on airfoils alone.

PUBLICATIONS RECEIVED.

Kolloidchemie. Ein Lehrbuch von Richard Zsigmondy. Fünfte, vermehrte und vollständig umgearbeitete Auflage. Zweiter spezieller Teil. 256 pages, illustrations, 8vo. Leipzig, Otto Spamer, 1927. Price, in paper, 14 marks.

Overhead Systems Reference Book, prepared by a special committee of the Overhead Systems Committee Engineering National Section National Electric Light Association 1927. 572 pages, illustrations, quarto. New York, National Electric Light Association.

A Guide to the Literature of Chemistry, by E. J. Crane, Editor of *Chemical Abstracts,* and Austin M. Patterson. 438 pages, portraits, plates, 8vo. New York, John Wiley and Sons, Inc., 1927. Price, $5.

Lubricating Greases, by E. M. Klemgard, B.Sc. 198 pages, 8vo. New York, The Chemical Catalog Company, Inc., 1927. Price, $5.50.

Laboratory Manual of General Inorganic Chemistry, by M. Cannon Sneed and Raymond E. Kirk. 181 pages, illustrations, 12mo. Boston, Ginn and Company, 1927. Price, $1.20.

Modern Eclipse Problems, being the Halley Lecture delivered on 20 May, 1927, with notes for the observation of the eclipse of 29 June, 1927, by F. J. M. Stratton. 34 pages, plates, 8vo. New York, Oxford University Press, 1927. Price, 85 cents.

Standards and Tests for Reagent and C.P. Chemicals, by Benjamin L. Murray. Second edition, revised and enlarged. 560 pages, 8vo. New York, D. Van Nostrand Company, 1927. Price, $5.

Chemical Engineering Catalog, 1927 (twelfth annual) edition. Collected, condensed and standardized catalog data of equipment, machinery, . . . with a classified index . . . and a technical and scientific book section. 1160 pages, illustrations, quarto. New York, The Chemical Catalog Company, Inc. Copies will be loaned to chemical engineers, chief chemists, heads of chemical engineering departments in universities, colleges and technical schools, technical departments of governments, and libraries with the understanding that they will be returned on publication of the next succeeding edition or they may be purchased outright by any of the above for three dollars. Price to those not included in any of the above classes, ten dollars.

National Advisory Committee for Aeronautics. Twelfth annual report, 1926, including Technical Papers Nos. 233 to 256. 631 pages, illustrations, quarto. Washington, Government Printing Office, 1927. Price, in paper, $1.50.

National Advisory Committee for Aeronautics: Technical Notes, No. 263, A Load Factor Formula, by Roy G. Miller. 8 pages, illustrations, quarto. No. 264, Tests of the N. P. L. Airship Models in the Variable Density Windtunnel, by George J. Higgins. 8 pages, illustrations, photograph, quarto. No. 265, Measurement of the Moments of Inertia of Full-scale Airplanes, by M. W. Green. 13 pages, illustrations, photograph, quarto. No. 266, Airfoil

Lift with Changing Angle of Attack, by Elliott G. Reid. 15 pages, illustrations, photograph, quarto. Washington, Committee, 1927.

The Mutochrome, for the production of coloured design in the following trades: Calico printing, textile manufacture, wallpaper manufacture, linoleum manufacture, carpet weaving, furnishing, colour printing, poster design, ceramics, decorative design, dressmaking and millinery, lithography, shop window advertising. Made by Adam Hilger, Ltd. 15 pages, illustrations, colored plates, quarto. London, Adam Hilger, Ltd., 1927.

Lehigh University Publication No. 7, Institute of Research, Circular No. 10. Methods for Exciting and Calibrating Tuning Forks, by Elias Klein, Ph.D., and Glenn F. Rouse, Ph.D. 24 pages, illustrations, 8vo. Bethlehem, Penna., Lehigh University, 1927. Price, fifteen cents.

CURRENT TOPICS.

The Evaporation of Water and Salt Solutions from Surfaces of Stone, Brick and Mortar. A. P. LAURIE and JOHN MILNE. (*Proc. Royal Soc. Edinburgh*, **47**, Part I.)—" It is generally accepted that one of the main causes of the rapid decay of stone and brick under modern conditions is the crystallization of calcium sulphate below the surface, causing a break-up of the stone and brick." Four specific cases are given in which calcium sulphate was found to be responsible for the bad condition of the building material. The SO_2 of this compound could be traced to the combustion of coal but the calcium oxide was not contained in sufficient quantity in the stone or brick and had to be attributed to the transfer of soluble salts from the mortar to the adjacent parts of the structure. Such an action does not always occur since there are miles of sandstone houses in Edinburgh and other miles of brick houses in London that are lasting well. The problem which the authors set themselves is to find under what conditions calcium salts travel from the mortar into the stone and brick, there to cause disintegration.

To obtain the basic and quantitative facts, analysis was made of the material of a brick taken from a building about thirty years old and showing decay at its surface. When the specimens analyzed came from parts of the brick away from the face and also from mortar almost no SO_3 was found and from 1½ to 2¼ per cent. of CaO. On the other hand, scaly and crumbling surfaces showed as much as 10 per cent. CaO, 25 per cent. $CaSO_4$ and 17 per cent. SO_3. " A white efflorescence contained 36.23 per cent. of calcium sulphate." Mortar located from the face of the building to points two inches back had the following composition: Moisture, 2.1 per cent.; silica and insoluble, 83.2; calcium carbonate, 10.9; calcium sulphate, .37, and free lime none. The small percentage of calcium sulphate in the mortar is noteworthy.

Analyses were made of samples of the brick taken .5 cm. from the edges that were in contact with mortar and from a hole midway between these two edges. The results arranged thus, edge, middle hole, edge, are as follows: CaO, 1.9 per cent., 1.4 per cent., 2.0 per cent.; SO_3, 1.4 per cent., 2.1 per cent., 1.8 per cent. " It will be seen that there is a higher percentage of lime in excess of that required to form sulphate of lime present in the drills near the edge than in the centre drill, indicating the passage of lime from the mortar into the brick."

Experiments were then made to discover whether there is transfer of liquid from brick to mortar or from mortar to brick when a brick wall has been rained upon and subsequently dries out. Blocks of porous sandstone were taken and treated in whole or in part with silicon ester in order that the deposited hydrated silica

might change the size of the interstices and consequently the conditions of evaporation if the sandstone were wet. By the use of potassium sulphocyanide and iron salts as indicators it was established that "when, therefore, a porous sandstone is drying, if one portion evaporates more slowly than another portion there is a steady flow of water from the slow-drying to the quick-drying portion." This result was confirmed by quantitative measurements of the amount of NaCl transferred from one part of the stone to another evaporating at a different rate. By coating the lateral but not the end surfaces with vaseline, it was established that the motion of the salt solutions did not take place over the surface of the stone but through its mass.

Similar results were obtained with bricks instead of porous stone. The conclusion was reached that, in the case of the brick structure examined, which seemed free from defects in construction or in the bricks or mortar used, the sulphuric acid came from the air and rain and united with the lime found chiefly in the mortar. Thus calcium sulphate was formed. During the process of evaporation this salt passed in solution from mortar to brick, where it crystallized and caused disintegration, the transfer of the solution to the brick being dependent on the more rapid evaporation of water from the brick than from the mortar.

A study of the conditions affecting local crystallization follows. "There is little doubt that the decomposition of stone in this country (Scotland) is due to the long periods in which the atmosphere is more or less saturated with moisture, and it is also true that in most cases the decay is found to be more rapid on surfaces which are protected from the sun and wind, such as under cornices and such like places where they are damp for long periods. The right way to prolong the life of stone buildings in this country is to hose them down at intervals during the hot weather, especially under the cornices, so as to promote surface crystallization of salts and the washing of them from the surface." G. F. S.

Remarkable Effect of Sunlight on the Height of the Aurora Borealis. CARL STÖRMER. (*Comptes Rendus*, July 25, 1927.)—On the evening of September 8, 1926, the author at Oslo observed an auroral drapery, violet-gray in color, extending from the western horizon upwards. This display was seen at the same time from the Shetland Islands. A series of photograms of the aurora was made in Norway from points 26 km. apart, from the working out of which it appears that the drapery extended from a level 300 km. above the earth to another 500 km. above it. The surprising feature of this conclusion is its contrast with the altitudes of from 80 to 200 km., which was the range for all auroras photographed in Norway from 1911 to 1922. Later in the night the drapery changed into a diffuse form extending above the altitude of 1000 km.

Störmer had reason to make the supposition that the drapery,

high over the earth and in the west toward the sun, was in the sunshine. This was verified by calculation. Subsequent comparison of other unusually high auroras showed that they also had been exposed to sunlight. Rays with their lower edges 300 to 400 km. above the earth and their highest points at an elevation of 600 or 800 km. were illuminated by the sun throughout, 95 to 100 per cent. of the ordinary rays seen at Oslo from 1911 to 1922 were in darkness and had no higher range than from 100 to 400 km. The high auroras have generally red, blue or violet as colors, while the lower ones are a greenish-yellow. The spectroscopic study of the high auroras may furnish very valuable data on their origin and on the conditions holding in the upper atmosphere. G. F. S.

The Transparency of Amorphous Silica for Ultra-violet Radiations. TSUKAMOTO. (*Comptes Rendus*, July 4, 1927.)—This material is coming into considerable use on account of its transmitting short wave-lengths, yet the data on this property are discordant. Shenstone, for example, says it is as transparent as crystalline quartz, while Pfluger states that 2 mm. of it absorbs 52 per cent. of radiation of wave-length 2400 A.U. and lets practically nothing through below 2200.

Tsukamoto had his experimental material prepared by melting quartz that was quite clear. After melting and cooling, a mass was obtained homogeneous in appearance but containing a few bubbles. From this was cut a piece with parallel faces 10 mm. apart. He found no difficulty in photographing through this plate the 1862 line of aluminum. By photographic photometry he determined the fraction of the incident light absorbed by 1 cm. of the material, and the absorption coefficient. For wave-length 1862 A.U. these quantities are, respectively, 83.6 per cent. and 1.81 cm. As longer wave-lengths were tried it was seen that the per cent. of light absorbed diminishes little up to 1930 A.U. while farther on the change is more rapid. At 2182 A.U. 33 per cent. is absorbed and the coefficient is .40 only. This artificial plate absorbed much more strongly than natural quartz. At 1862 the latter absorbed but 25 per cent. in contrast to the 83.6 per cent. absorbed by the plate, but this was true only for the most transparent specimens. G. F. S.

The Constitution of Ordinary Lead. F. W. ASTON. (*Nature*, Aug. 13, 1927.)—Soddy, a decade or two ago, ventured the opinion that the atomic weight of lead would be different according to the kind of lead used, whether it be ordinary lead or the same metal resulting from the disintegration of uranium or of thorium. Careful experiments later confirmed this prediction. In comparison with 207.2 as the atomic weight of common lead, an atomic weight as low as 206.05 was found for lead of uranium ancestry and another as high as 207.9 for thorium lead.

Aston, whose lectures at The Franklin Institute a few years ago attracted much attention, has at last obtained the mass-spectrum of lead by the use of its tetramethyl compound. The vapor was used either diluted with carbon dioxide or pure. Long photographic exposures were required. "The three principal lines are 206 [4], 207 [3], 208 [7]. The figures in brackets indicate roughly the relative intensities and are in good agreement with the atomic weight 207.2. There are indications that many other isotopes may be present in small proportions. An exceedingly faint line at 209 occurs in the atomic group and on one plate is visible on the $PbCH_3$ group. This is almost certainly an isotope." Thus by an additional method the composite character of lead has been proved.

In the same experiments the mercury lines were stronger than in any antecedent investigation so that a seventh isotope of this metal was determined with certainty, the atomic weight being 196. It is present to the extent of only .06 per cent. G. F. S.

Electric Conductivity at Low Temperatures. J. C. MCLENNAN and C. D. NIVEN. (*Phil. Mag.*, Aug., 1927.)—"It has been known for a long time that the electrical resistance of many pure metals is approximately proportional to the absolute temperature. At low temperatures results have shown that this law is very far from being true. In the case of some metals the resistance suddenly vanishes altogether before the absolute temperature (zero?) is reached—a phenomenon called by the late Prof. H. K. Onnes superconductivity. In the case of other metals the resistance often tends to a constant value. Work by Onnes and his collaborators at Leiden, and Meissner at Charlottenburg indicates that superconductivity is a property of only five metals, namely, mercury, lead, tin, thallium and indium." The possession of a plant for making helium by the University of Toronto made it possible for the authors to conduct experiments on resistance in the vicinity of the absolute zero, a thing that can be done in but few laboratories. Those who attended Section A of the British Association at Toronto in 1924 will recall the display of liquid helium there made.

Resistances were measured at the temperatures of the room, of liquid air, liquid hydrogen and liquid helium. "Temperatures below that of liquid helium were obtained by boiling the liquid helium under reduced pressure, the pressure on the boiling liquid giving the temperature by reference to the vapour-pressure curve." A Mueller temperature bridge and a Cambridge Instrument Company's vernier potentiometer were used to measure the resistances. To test the method and apparatus, measurements were first made on metals—lead, cadmium, indium, sodium and potassium—whose resistances had previously been examined at very low temperatures by other investigators.

The lead for measurement was got by heating to a white heat a strip of molybdenum in a vacuum. Lead resting on the strip was

thus vaporized and, ascending, deposited itself on the under surface of a horizontal ebonite plate. A current of 35 amperes for one minute gave a satisfactory deposit. Lines were ruled through the film of deposited lead producing a zigzag ribbon having about 18 ohms resistance at 0° C. This ribbon, after having been coated with collodion to prevent oxidization, remained unchanged for months. The resistance of this lead film decreased linearly from 17.93 ohms (specific resistance = 20.4) at 0° C., or 273° Kelvin, to 5.09 ohms (sp. res., 5.79) at 82° Kelvin, but at the temperature of liquid helium it became zero. "The question then remained to be answered, did the resistance gradually diminish according to the linear relation or did it suddenly disappear as Prof. H. K. Onnes had found for a lead turning?" This point was examined with great care, and the following resistances were found at the Kelvin temperatures stated, 20.6°, .87 ohm; 14.4°, .54 ohm; 10.3°, .40; 7.6°, .35; 7.5°, .34; 7.4°, zero. The needle of a millivoltmeter fell steadily back as the temperature descended until at about 7.5° it suddenly fell to zero. Onnes found the temperature for the vanishing of resistance for lead to be 7.2°.

The resistance of cadmium was measured down to 3.6° without any evidence of superconductivity appearing. There seems to be almost no change of resistance from 8° to 3.6°, this residual resistance being only 7.8×10^{-4} of the resistance at zero. For this ratio Meissner gives 7.6×10^{-4}. The Toronto physicists found indium to become superconducting at 3.38°, whereas Onnes gives it as 3.42°. This close agreement is indicative of the high accuracy attained in this difficult field of research.

The resistance curves of beryllium, chromium, rubidium, and thorium, as well as of an alloy of sodium and potassium, were traced for the first time to the vicinity of 0°. Beryllium at 4.2° retained 78 per cent. of its resistance at 293°. Chromium displayed large differences in resistance in consequence of the heat treatment applied. The Smithsonian tables give for its specific resistance 2.6 microhms, but the authors got a value about sixteen times as great. Beryllium, chromium and rubidium betrayed no sign of superconductivity. The specific resistance of an alloy of sodium and potassium departs widely from the temperature relations of its constituents from 273° to 192°. Below this temperature its curve, like those of its constituents, is almost a straight line. The alloy always has a much larger specific resistance than either sodium or potassium.

G. F. S.

The Surface Tension of Bismuth, Cadmium, Zinc and Antimony. L. L. BIRCUMSHAW. (*Phil. Mag.*, June, 1927.)—The surface tension of these metals in the molten state was measured by getting the pressure developed in bubbles of inert gas blown from orifices immersed in the liquid. For bismuth the temperature range was from 300° C. to 962°. This considerable elevation of tempera-

ture reduced the surface tension from 388 dynes/cm. only to 340. Cadmium strangely enough has a maximum value at 425° C. Of the metals examined molten zinc possesses by far the highest surface tension. G. F. S.

The Surface Tension Balance. E. L. WARREN. (*Phil. Mag.*, Aug., 1927.)—"Surface-tension measurements play an important part in modern colloidal work, and for this reason it has become necessary that methods for their determination should be developed which are accurate, rapid, and do not make too great a demand on the instrumental equipment of a technical laboratory. The capillary-ascent method certainly does not satisfy these requirements." Capillary tubes of uniform bore are hard to find. They are hard to clean and to keep clean and troublesome to calibrate. It is not easy to measure accurately the height of the ascent of the liquid. The temperature of the meniscus requires elaborate arrangements for its measurement. Even after the overcoming of all these difficulties the uncertainty of the value of the angle of contact between the liquid and the material of the tube remains to vitiate the results. Good authorities are by no means in agreement on the magnitude of the contact angle between water and glass, for example. Quincke gives it as 8–9°, while Wilberforce holds that it is 0°.

The surface tension balance embodies a modification of Jaeger's method and was devised in order to furnish a means of measuring the temperature coefficient of surface tension and of studying the dependence of the surface tension of solutions upon their concentration. Its essential feature is that it compares the surface tension of the experimental liquid with that of a standard liquid such as water. A bottle contains air at a pressure of a few mm. of mercury above that of the outer air. A vertical glass tube rises from this bottle, is bent horizontally and enters a rubber tube provided with a pinch-cock. From the other end of the rubber tube a horizontal glass tube proceeds. From this stretch downward into two beakers two vertical glass tubes having at their lower ends tapering parts that end in holes exactly alike. This is secured by drawing out a glass tube, cutting it perpendicular to its length and by using the two ends previously united for the lower terminations of the vertical tubes. The two beakers contain different liquids covering the ends of the vertical glass tubes. The pinch-cock is opened until bubbles of air break away at intervals of about five seconds from the open tube in one of the beakers that is fixed in elevation. The other beaker is then adjusted vertically until the height of its liquid above the orifice of its inserted tube is just sufficient to cause bubbles to issue alternately with those forming in the other beaker. The difference between the surface tensions of the two liquids in which these equal bubbles are forming under like conditions is shown to depend upon their densities and the vertical distance from the surface of the liquid in the beaker to the lowest point of the meniscus at the moment when the

bubble breaks away, as well as upon the acceleration of gravity and the radius of the aperture, which is the same in both beakers.

To study the change of the surface tension of water with temperature the beaker of fixed position was covered with asbestos to prevent change of temperature. In it was put pure distilled water whose surface tension at 15° was taken at 73.65 dynes per cm. as the mean of the results of thirteen previous measurements made by others. The beaker of adjustable height likewise contained water and was surrounded by a heating coil. The temperature was raised by 5° intervals and the balance of bubble formation was obtained at each temperature. From the observations the surface tension of water is calculated from its freezing-point to 90°. The values at these two temperatures are 75.94 and 60.80 dynes/cm. It is stated that these results as well as those at intermediate temperature are correct to .005 dyne/cm. or to less than 1/100 per cent., provided, of course, that the value assumed for 15° is correct. "It was difficult to take readings above 90° C. because the evaporation then becomes very rapid, and furthermore the residue of dissolved air is evolved and disturbs the surface."

The same method was used to investigate the change in the surface tension of sodium chloride solutions due to increasing concentrations, ranging from 12½ to 300 grams/litre. At 20° C. the surface tension is 73.41 for a concentration of 12½ grams/litre and rises to 82.53 for 300 grams/litre. G. F. S.

Unverdorben, the Discoverer of Aniline.—LUDWIG DARMSTAEDTER, of the Prussian State Library at Berlin (*Ind. Eng. Chem.*, News Edition, 1927, **5**, no. 19, 9), gives a concise account of the discovery and history of aniline. Otto Unverdorben was born at Dahme in Brandenburg, October 13, 1806, and died there, December 27, 1873. He was a pupil of Johann Trommsdorff at Erfurt and of Heinrich Rose at Berlin. In 1826, Unverdorben subjected indigo to destructive distillation, and obtained among the products a base which he designated "krystallin." In 1837, Runge discovered "kyanol" in coal tar. In 1840, Fritzsche produced "anilin" by heating indigo with potassium hydroxide at a temperature above 150° C. In 1842, Zinin reached an ammoniacal solution of nitrobenzene in alcohol with hydrogen sulphide, and obtained "benzidam." In 1843, Hofmann repeated all four procedures, and demonstrated that they yielded the same product. Berzelius objected to the name "crystalline" as a general characteristic of compounds, so the name "aniline" was adopted. In 1856, Perkin discovered mauve, the first aniline or coal-tar dye. Hofmann produced aniline red in 1857 and his violet in 1863, while Lightfoot prepared aniline black in 1862. The demand for aniline could not be satisfied by the preformed aniline of coal tar. Synthetic methods were devised and worked on a commercial scale; they were based on the reduction of nitrobenzene; Hofmann used zinc and hydrochloric acid. In 1854,

Bechamp introduced the use of iron borings and acetic acid. In 1865, Brimmeyer modified Bechamp's process by substitution of hydrochloric acid for acetic acid; this method had become the general practice in 1866. In 1869, the production of aniline was approximately 1,500,000 kilograms. J. S. H.

Structure of the Carbohydrates.—In his Willard Gibbs address, JAMES COLQUHOUN IRVINE, of the University of St. Andrews (*Chem. Rev.*, 1927, 4, 203–229), reviews his researches on the structure of the carbohydrates made during the past quarter century. In this study use has been made of the methyl derivatives of the carbohydrates; the compounds, which have been of greatest service, are 2, 3, 4, 6 tetramethylglucose, 2, 3, 6 trimethylglucose, 2, 3, 4 trimethylglucose, tetramethyl gamma fructose and 2, 3, 4, 6 tetramethylgalactose. In glucose, an oxygen ring may extend from the terminal aldehyde (which thereby becomes a CHOH group) to any of the five remaining carbon atoms; an aldehydic variety is also possible; therefore *d*-glucose may exist in eleven forms, and may react as a mixture of any of these. Structural formulæ have been derived for the disaccharides cellobiose, isocellobiose, maltose, isomaltose, cellobiose, isocellobiose, melibiose, gentiobiose, and sucrose. The fructose segment of sucrose is gamma fructose. The structural formula has also been derived for the trisaccharide, raffinose, which contains a glucose residue linked on the one hand to a galactose residue, and on the other hand to a fructose residue. Celluloses apparently are polymerides of several basal units; each unit contains several chains of six carbon atoms each; in some units the number of chains present is even, in other units it is odd. Each chain is a glucose residue; the residues are linked always in an identical manner; and one of the units is anhydrotriglucose. Starch contains anhydrohexaglucose as a molecular unit. Insulin is a polymeride or series of polymerides of anhydro gamma fructose. J. S. H.

The Rusting of Steel Surfaces in Contact. G. A. TOMLINSON. (*Proc. Roy. Soc.*, A 771.)—Two hard steel surfaces pressed against each other with some force and having relative motion after some time show a brown discoloration at the contact surface. "When two machined steel surfaces are held firmly in contact and at the same time are subject to vibration, it is often found that on taking them apart that the surfaces have become cemented together by the production of relatively large quantities of oxide, and the individual surfaces are badly pitted and have a corroded appearance. Oiling of the surfaces before assembly certainly does not prevent this effect and there is little direct evidence as to whether it is beneficial to a limited extent or not at all." This effect was discussed in the *Engineer* in 1911 but since that time it seems to have attracted no further notice.

The experiments reported in this paper were carried out at the

National Physical Laboratory of Great Britain. First a steel ball was rotated on a slip gauge. Both were of hardened steel and their surfaces were cleaned mechanically. The load was 13 oz., which for the surface of contact produced a pressure of 223,000 lbs. per sq. in. After eight revolutions of the ball the unaided eye could see a minute brownish spot that resisted removal by a cloth. The photograph reproduced in the paper shows that "a single passage of one surface over the other is sufficient to produce a track of oxidation débris that is plainly visible." With the same arrangement of ball and surface but with a pressure only 13 per cent. as great no rust was obtained. It seems that some kind of a film kept the surfaces apart, the applied pressure being inadequate to rupture it. A larger steel ball .469 in. in diameter weighted with only .23 gm. was drawn to and fro 100 times over a steel surface. Both contact surfaces had been cleaned chemically so that any film was probably removed. The pressure was 5000 lbs./sq. in. Considerable brownish material resulted. Since in this experiment the surfaces in contact were so highly polished that no scratches were seen under the microscope, it was impossible that by mutual grinding fine metallic powder was produced which afterwards oxidized to form the brown material.

Relative tangential motion of the two surfaces in contact is necessary for the production of the brown stain. This was proved by two experiments. A small steel ball was pressed against a surface of the same material and let remain for forty hours. There was no motion but the pressure amounted to 354,000 lbs./sq. in. No brownish material appeared. Again two steel balls rolled backward and forward over a steel surface several thousand times without the formation of brown débris. In pure rotation such as this there is no relative tangential motion at the point of contact. When polished glass, stellite or agate spheres were moved to and fro over chemically cleaned steel surfaces, marked brown residue was obtained, though the pressures were rather small. "Experience with ball-bearings proves that the rusting effect does not occur with pure rolling."

The author explains the effect by the detaching of molecules through the application of tangential force and their subsequent oxidation by the air. To account for the inability of molecular cohesion to extract molecules from either surface when they separate after intimate contact, he advances the opinion that the force which acts between two molecules of different bodies that approach each other in a direction perpendicular to the surface of separation is smaller than the force binding either to the other molecules in the same body. "The nature of the bonds between the molecules appears to be such that the cohesive force of a visiting molecule is quite insufficient to pluck the molecule out normally, but is sufficient to detach it from the solid when applied tangentially. To use a crude analogy, a tooth is more easily uprooted by a side pull than by a

normal pull." In proof of the readiness with which cohesion takes place he cites these experiments. "If a piece of plate glass and a fused glass bead at the end of a rod are both carefully cleaned, and the rod is poised lightly in the fingers and allowed to stroke the plate, there is felt to be a sequence of snatches as the bead welds on and breaks away repeatedly. If afterwards the plate is examined with a lens, it is seen to be scored with fine dotted lines wherever the bead has touched it, even with the lightest touch. In the same experiment with hard steel surfaces the tactile sensation of welding is less marked, but if the plane surface is examined under the microscope the path followed by the ball is shown by a thin track of reddish oxidation débris." This last experiment shows that particles, whether molecular or larger in size, are torn off by tangential motion and rapidly oxidized. Since the range through which molecular forces act is very small, only a very small tangential motion should suffice to detach molecules according to theory. This is confirmed by the following experiments. In the first, repeated contacts were made between a spherical face and a plane polished steel surface. All contacts were at the same point. There was probably a slight lateral motion due to elastic deformation when the load was applied. In any case the oxidation effect showed itself with steel, glass, agate and stellite spheres. In the second a steel ball was made to swing backwards and forwards over a distance of 8.5×10^{-8} inches, remaining in contact with a steel plate. Fifty thousand oscillations were executed. A small quantity of oxidation débris was found at the contact point. Smaller amplitudes were tried, viz., 6.5×10^{-8} and 3.5×10^{-8} inches. With the smaller of these amplitudes no rust was found, while with the larger a discoloration was found with difficulty. Attention is directed to the agreement of this result with the value of the range of molecular attraction given by Johonnot and Chamberlain independently as 8×10^{-8} inches. "These results also make it clear how copious rusting may occur at certain contact surfaces in high-speed machines, though apparently the surfaces are secured rigidly together. Actually there is probably a minute to-and-fro motion occurring as the rust is invariably found associated with vibration. With high-frequency vibration the aggregate motion per hour may be considerable, hence the large accumulation of oxide found after a length of time."

In the opinion of the author his experiments are incapable of interpretation on the basis of electrolysis, adsorption or local heating as producing the rusting effect.

Further experiments are given in which steel spheres moved over polished steel planes, the surface being lubricated with vaseline or castor oil. No rusting was found with small pressures, but it appeared with higher pressures. This change in effect is attributed to the rupturing of the film of lubricant by the high pressure and to the consequent approach of the steel surfaces. G. F. S.

Food Products.—In his report on food and drug inspection and analysis, E. M. BAILEY (*Connecticut Agric. Exp. Station Bull. 287*, 361–391, 1927) calls attention to the use of hydrogen peroxide as a preservative in bottled chocolate beverages. A sample of dipped eggs (eggs preserved by dipping in oil) was wholesome and edible, but lacked the characteristics of fresh eggs with the sole exception of fairly small air spaces. The eggs had a relatively high ammoniacal nitrogen content, and their yolks had settled in the shells. The chemical composition of shelled acorns varied with the species of oak. The acorns of a given species underwent changes in chemical composition when stored in earth during the winter. J. S. H.

Hydrogen-ion Concentration of Solutions of the Carbonates of Calcium.—According to JOHN WARREN WILLIAMS and JOSEPH A. CHUCKA (*Jour. Am. Medical Asso.*, 1927, **89**, 445–446) a saturated aqueous solution of calcium carbonate at 25° C. has a hydrogen-ion concentration of pH 8.8. A solution of calcium bicarbonate free from an excess of carbon dioxide has a hydrogen-ion concentration of pH 6.2; a solution of this salt, through which carbon dioxide is passing and which is saturated with that gas, has a hydrogen-ion concentration of pH 6.0. J. S. H.

Vitamins.—JOSEPH GOLDBERGER, G. A. WHEELER, R. D. LILLIE and L. M. ROGERS (*Public Health Reports*, 1926, **41**, 297–318) find that the pellagra preventive factor, designated $p - p$, is not present to any appreciable extent, if at all, in butter, but is present in fresh lean beef and in dried yeast extract. $p - p$ is distinct from vitamin B, the anti-neuritic vitamin, and is thermostable, surviving heating in an autoclave. The diseases, pellagra in man and black tongue in dogs, are recognized as analogous. GOLDBERGER and LILLIE (*Public Health Reports*, 1926, **41**, 1025–1029) have observed a pellagra-like condition in rats whose diet lacks $p - p$. This condition in the rats is cured by addition of $p - p$ to the ration. MAURICE I. SMITH, W. T. MCCLOSKY and E. G. HENDRICK (*Public Health Reports*, 1926, **41**, 767–779) find a deficiency of vitamin B in the ration renders rats more susceptible to pilocarpine and ergotoxine, while a deficiency of vitamin A renders them more susceptible to ergotoxine and morphine and, to a slight extent, to histamine, ouabain, strychnine, atropine, cocaine, and apocodeine. In other words, a deficient supply of vitamins renders the animal body more susceptible to the action of certain poisons. J. S. H.

Biochemistry of the White Race in the Tropics.—E. S. SUNDSTROEM (*Univ. California Pub. Physiol.*, 1926, **6**, 1–216) has made a series of contributions to tropical physiology with special reference to the adaptation of the white man to the climate of North Queensland. The fasting *blood sugar* of 45 individuals in the tropics

varied between 86 and 145 mg. per 100 c.c.; the tendency was for a lower value in women than in men, and for a slightly lower value in individuals of both sexes who had lived in the tropics since birth than in immigrants. A hot and humid climate predisposed to a lowering of the blood-sugar level, although this level may depend partly on the maintenance of normal body temperature. The *total non-protein nitrogen* of the blood in these subjects varied between 29 and 57 mg. per 100 c.c. of whole blood. Women usually gave lower values than men. The tropical environment apparently predisposed to a relatively high level of total non-protein nitrogen, while its partition as urea, uric acid, creatine, and creatinine remained normal. Their retention was probably due to lack of water for their proper excretion. Seasonal variations occurred in the total non-protein nitrogen and *urea* of the blood, minimum values occurred during the cooler, and maximum values during the hotter part of the year. *The acid-soluble phosphorus* of the *plasma* in the tropics agreed fairly well with normal standards. Its *lipoid phosphorus* remained normal in males, but was low in females. Both forms of phosphorus in the *corpuscles* remained normal only during a short tropical residence, they then decreased; this decrease in acid-soluble phosphorus was due to a decrease in organic compounds containing phosphorus. Both forms of phosphorus in the *whole blood* were normal during a short tropical residence, then decreased to average values of 11 to 16 mg. of acid-soluble phosphorus per 100 c.c. blood, and 9 mg. (in men) and 7 mg. (in women) of lipoid phosphorus per 100 c.c. blood. The level of acid-soluble phosphorus apparently depended on the sex and the period of tropical residence. The *total fatty acids* were considerably lower in the blood of tropical residents than in residents of cooler climates, this was especially true of females and of descendants of settlers. The *cholesterol* content of the blood had a somewhat wider range in the tropics than in cooler regions, but no other decided differences were noted. The *fat* content of the corpuscles tended to be high when their phosphatide content was abnormally low. The ratio, lecithin: cholesterol, ranged between 0.84 and 1.81 in men and between 0.60 and 1.34 in women, wider ranges than occur in cooler climates; subnormal ratios were exhibited chiefly by women and by men born in the tropics. The lipoid levels were influenced by the season of the year. The *suspension stability* of the *erythrocytes* (red blood-cells) tended to decrease during the hot season and to return to normal in cooler weather. In the tropics, the *total solids* and the *chlorides* of the *whole blood* increased; the resistance of the *erythrocytes* to hypotonic saline solutions apparently increased; and measurement of their diameters indicated a relative preponderance of large red cells. Stress is placed upon the necessity of an intentionally augmented water intake in the tropics. The *alveolar carbon dioxide tension* was lowered and the pH of the blood changed to a higher level in the tropics; this evidence of alkalosis was supported by the *oxyhemoglobin dissociation curve*.

The alkalosis was reduced by copious ingestion of water. The *basal metabolism* in a tropical region, at a time when the most oppressive weather was past, ranged between 25.5 and 36.1 with an average of 31 Cal. per m^2. per hr. in men, and between 22.1 and 33.8 with an average of 27.8 Cal. in women. A relatively high value was obtained in physically active individuals. The humidity tended to influence the result. The daily *caloric intake* (studied for a week) in the tropics averaged 3044 Cal. for a man, 2374 Cal. for a woman, and 2078 Cal. for a girl of nine years; their average daily water intake was: 2650, 1730, and 1760 c.c., respectively. The *nitrogen balance* showed a loss of 0.15 g. by the man, and retention of 0.56 g. by the woman and of 1.28 g. by the girl, who still showed retention after correction for cutaneous loss of nitrogen. The *phosphorus balance* was positive and the *sulphur balance* negative in all three subjects. The child showed a definite retention of *alkalies* equal to approximately 200 c.c. 0.1 *N* solution. The adults exhibited an almost complete *acid-base equilibrium*, apparent retention of chlorine and alkalies being covered by loss of sodium chloride through the skin. The *urine* in the tropics was normal with respect to its pH range titratable acidity, "normal sugar," neutral sulphur, and creatinine content, the values being similar to those found in temperate regions. Its ammonia coefficient tended to be somewhat high. Adolescent white school girls in the tropics *grew* more rapidly but did not *mature* earlier than girls in cooler parts of the same regions of the earth. The *hair* and *nails* grew about 10 per cent. less rapidly in the tropics. *Circulation of the air* did not increase the growth rate of rats, but increased the weight of their viscera. In the tropics the peripheral blood tended to exhibit an increase in the *erythrocyte count*, decreases in the *leucocyte count* and the relative number of *polymorphonuclear cells*, and a "shift to the left" of the *Arneth count*. The tropical climate did not affect the *reaction time* to visual stimuli. Many of the results obtained on human beings in this series of researches were confirmed by the results of similar experiments on rats. J. S. H.

Radio-activity of Natural Waters.—W. D. COLLINS, of the U. S. Geological Survey (*Public Health Reports*, 1926, **41**, 1937–1939), states that every natural water is more or less radio-active, and the presence of a small amount of radio-active matter in a spring water does not set that water apart from other natural waters. The radium emanation is approximately half gone in about 3.8 days after the water has been taken from its source and practically all gone within thirty days. In order to obtain a therapeutic dose of radium emanation, it would be necessary to drink from 100 to 1000 gallons a day of a naturally radio-active spring water! The small amount of radio-activity in natural waters does not have any effect on the medicinal value of the waters. J. S. H.

William Hyde Wollaston.—Since the year 1928 will be the centenary of the death of William Hyde Wollaston, H. G. WAYLING (*Science Progress*, 1927, **22**, 81–95) has written a short account of his life and contributions to science. Born August 6, 1766, Wollaston was educated in medicine at Cambridge, and finally became president of the Royal Society. His researches covered a wide field in chemistry, physics and medicine, and included such subjects as the cryophorus, the reflecting goniometer, the camera lucida, the dark lines in the solar spectrum, the power of frictional electricity to effect polar chemical decomposition, malleable platinum, palladium, rhodium, and urinary calculi. He died December 22, 1828. J. S. H.

Dr. L. O. Howard Resigns as Chief of the Bureau of Entomology. (Department of Agriculture.)—After more than thirty-three years of service as chief entomologist of the United States Department of Agriculture, Dr. Leland O. Howard retired on October 17th as the chief of the Bureau of Entomology, and was succeeded by Dr. C. L. Marlatt, a member of the department since 1888, and who for the past five years has been associate chief in charge of the regulatory work of the bureau, and also chairman of the Federal Horticultural Board.

Doctor Howard is now in his fiftieth year of government service, having joined the entomological branch of the Department of Agriculture in 1878, soon after his graduation from Cornell University. He retires as chief at his own request, but this does not mean retirement from service. He has passed his seventieth birthday, and has asked to be relieved of the administrative duties of his office, but proposes to devote his full energies to the field of entomological research in which he has long been recognized as perhaps the most distinguished investigator. His favorite fields are medical entomology and parasitology.

Doctor Howard was placed in charge of the entomological work of the department June 1, 1894. In the thirty-three years that have followed, the science of entomology has broadened tremendously and Doctor Howard has guided numerous activities which have been of great service to the American public.

Two campaigns with which Doctor Howard has been identified have captured the public fancy. He was a leader in the mosquito crusade. As early as 1892 he published results of experiments showing that certain types could be controlled by the use of kerosene, and when the mosquitoes were identified as disease carriers he was able to recommend methods of control. His publications on the house fly dating from 1896, to his book, "The House Fly Disease Carrier," in 1911, were largely responsible for the anti-house fly crusades all over the world in the last twenty years.

Doctor Howard is a member of the three great American associations of limited membership, the National Academy of Sciences, the

American Philosophical Society, and the American Academy of Arts and Sciences. He was Permanent Secretary of the American Association for the Advancement of Science for twenty-two years, and its president in 1920–1921. He has been made honorary member of many foreign scientific societies and is the only American honorary member of the Academy of Agriculture of France, and received several decorations among which are the Cross, Chevalier de la Legion d'Honneur, and the Cross, Officer de l'Odre du Merite Agricole. He has been a delegate to many international assemblies and an officer of six scientific gatherings. In addition to bachelor and master degrees from Cornell, his doctorates include Ph.D. (Georgetown, 1896), M.D. (George Washington, 1911), LL.D. (Pittsburgh, 1911), and Sc.D. (Toronto, 1920). The bibliography of his publications includes 941 titles.

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